Does Level-k Behavior Imply Level-k Thinking?

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Abstract

The level-k literature classifies subjects into different Lk types by the reasoning steps they use in the games. However, it remains unclear whether the observed level-k behavior is determined by belief or reasoning ability. This study identifies the decisive factors that prevent people from using a greater depth of reasoning. It distinguishes between the belief-bounded "Lkb" players, who have high reasoning ability and best respond to the belief that the opponents reason $k-1$ steps (Lk belief), and the ability-bounded "Lka" players, who could use, at most, k steps of reasoning. This separation utilizes a combination of simultaneous and sequential ring games. In the sequential games it requires more than k reasoning steps to respond to Lk belief, so Lkb players still best respond but Lka would fail. Results show that around half of the $L2$ and $L3$ subjects are belief-bounded, while the rest have reached their upper boundaries of reasoning. Additionally, subjects' CRT scores, a measure of their cognitive ability, support the separation of the two types. The findings suggest that both belief and reasoning ability could determine the observed levels, and thus one must be cautious when trying to infer belief or reasoning ability from the existing level-k data.

Keywords: behavioral game theory, level-k, high-order belief, bounded rationality
JEL Codes: C70, C91

1 Introduction

The level-k theory is proposed to model player's systematic deviation from Nash equilibrium by allowing for inconsistent beliefs (Nagel, 1995; Stahl and Wilson, 1994, 1995; Ho et al., 1998). The model starts with the irrational or non-strategic $L0$ type, who is usually assumed to choose randomly or use certain salient strategies. $L1$ acts as if all the rest of the players are $L0$ and

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each higher Lk best responds to $L(k-1)$. The level-k model has been used extensively to explain non-equilibrium behavior from labs and fields (See Crawford et al. (2013) for a detailed review).

Experimental literature on the level-k model mostly focuses on the identification of Lk types by looking at the number of reasoning steps used by the subjects. Most subjects in these experiments appear to be using no more than two or three steps of reasoning when playing a game for the first time. The natural question then is, why do they stop at these low levels? What can be inferred about people's reasoning process and reasoning ability in general?

To tackle this question, we need to understand what determines the observed levels. The original level-k model assumes that a subject using k steps of reasoning is responding to his belief that the opponent reasons $k-1$ steps. Here I call this type of belief the "*Lk belief*". A more rigorous definition of the belief space will be given in Section 3. However, observing reasoning steps alone is not sufficient to infer one's belief. A subject's exhibited sophistication level in a certain game depends not only on his belief about the opponents' reasoning levels, but also on his ability to finish all the required reasoning steps.

Therefore, using k reasoning steps implies two distinct cases about one's belief and ability. First, the subject could think more than k steps, and is best responding to Lk belief. In this case, the subject's behavior is determined by his belief and his reasoning ability is higher than the exhibited level. In the alternative story, the observed Lk behavior results from bounded reasoning ability. Even if the subject might believe that the opponents are more sophisticated than reasoning $k-1$ levels, his limited ability prevents him from developing too many thinking steps. Therefore, in the first case, k reflects the subject's belief level, while in the latter, k is the bound of his reasoning ability.

Disentangling the effects of belief and reasoning ability is relevant to a lot of research questions. For example, researchers are interested in using people's reasoning ability in one game to predict their performance in other games (Georganas et al., 2015) or the economic outcomes in the real world. Currently, these studies use observed levels as proxies of reasoning abilities, but more ideal analysis would be based on the identified ability bounds (perhaps together with identified beliefs). In addition, there are larger heterogeneities among the players from the real world than in the laboratories. Data on people's belief and ability bounds could help us predict their behavior when faced with more seasoned opponents, a situation always encountered in reality.

Evidence on belief and ability bounds could also add to our understanding of how people form beliefs. Do they form belief based on their own abilities? Or, are belief levels far below the subjects' ability bounds? In fact, previous studies that explore how levels are endogenously determined (Alaoui and Panta, 2016) and the correlation of levels between different games (Georganas et al., 2015) both apply models that distinguish between belief and reasoning ability. Therefore, separation of the two decisive factors could shed light on a more predictive theory stemming from the level-k model.

Nevertheless, since belief and ability are not directly observable from choice data, few studies have provided a clean identification of belief or ability bounds. This experiment attempts to identify whether subjects behave at the observed levels due to the belief that other participants do not think more than one or two steps, or due to their own lack of ability to think further. In this study, a Lk player is classified as Lkb (belief-bounded) if his observed level is determined by his belief, and as Lka (ability-bounded) if it is determined by his ability. The separation of Lkb and Lka players utilizes a combination of simultaneous and sequential ring games. The ring games are first studied in the innovative work of Kneeland (2015). She uses a set of simultaneous ring games to separate different orders of rationality. Here I provide an illustration of the separation of $L2b$ and $L2a$ with 3-player ring games.

Let us first consider a 3-player simultaneous ring game G (Figure 1). G is used to separate subjects with different reasoning levels, but G alone is not sufficient to separate belief-bounded and ability-bounded subjects. In this game, Player 1's (P1) payoff depends on his and Player 2's (P2) actions. Player 2's payoff depends on his and Player 3's (P3) actions. Player 3's payoff depends on his and Player 1's actions. In particular, Player 3 has a dominant strategy. Players 1 and 2's iterated dominant strategies could be derived from Player 3's best response, and thus the unique surviving profile $(b1, a2, b3)$ could be solved through three rounds of iterated dominance.

P1's Payoff

Player 1 \ Player 2	$a2$	$b2$
$a1$	0	10
$b1$	10	0

P2's Payoff

Player 2 \ Player 3	$a3$	$b3$
$a2$	0	10
$b2$	10	0

P3's Payoff

Player 3 \ Player 1	$a1$	$b1$
$a3$	0	0
$b3$	10	10

Note: In G, the three players move simultaneously. In G', players 1 and 3 move in the first stage. Player 2 moves in the second stage after observing players 1's and 3's choices.

Figure 1: G and G': 3-player ring games

G is similar to the ring games used in Kneeland (2015). In this game, Player 3 only needs to reason one step to play the dominant strategy. Player 2 needs to believe that Player 3 is best responding ($L2$ belief) and use two steps of reasoning in order to solve for his iterated dominant strategy. To best respond, Player 1 needs to perform 3 steps of reasoning and, at the same time, believe that Player 2 is at least $L2$ ($L3$ belief).

If a player consistently chooses the (iterated) dominant strategies as Players 2 and 3, but not as Player 1, he would be classified as $L2$. However, it could not be inferred from these simultaneous games whether he believes that Player 2 is no higher than $L1$, or whether he holds a higher-level belief but could do, at most, two steps of reasoning. The difficulty of identification lies in the fact that it always requires k steps of reasoning to respond to the Lk belief that opponents reason $(k-1)$ steps. Thus, it is hard to tell whether belief or reasoning ability is binding. This is also true in most games in the existing literature.

To cope with this problem, I introduce the sequential ring games, in which one needs to use more than k steps to respond to Lk belief. The sequential game G' adopts the same payoff structure as in G, but includes two stages. Players 1 and 3 move simultaneously in the first stage. Player 2 observes their actions and then moves in the second stage. The sequence of moves is common knowledge among all players. In this game, solving Player 3's problem still requires one step of reasoning. Since Player 2 directly observes Player 3's action, there is no need for him to reason through Player 3's decision problem. He also needs only one step to best respond.

The key to the identification lies in the Player 1 position of the sequential game G'. Obviously, Player 1 of G' still needs 3 steps to determine the best response "$b1$". But he only needs to believe that Player 3 picks the dominant strategy[1] and that, after observing Player 3's move, Player 2 best responds accordingly. So as Player 1 in G', even if it requires him to think 3 steps to best respond, he only needs to hold $L2$ belief.

Now consider a player who reasons two steps in the simultaneous game G. Such a player would be classified as $L2$ in the Lk literature. His behavior as Player 1 in G' reveals whether he is $L2b$ or $L2a$ (Table 1). If he believes that others are $L1$ and his ability is not binding, then he should believe that both Players 2 and 3 will be able to pick the dominant strategies, and choose his iterated dominant strategy as well (Row (3)). However, if he behaves like a $L2$ in G because he is bounded by two steps of reasoning, he would fail to obey iterated dominance as Player 1 of G' (Row (4)). Thus $L2b$ or $L2a$ could be separated using the Player 1 position of 3-player ring games. Larger rings are needed to get separation of higher types.

G: simultaneous	Player 1	Player 2	Player 3
(1) $L2b$	$\times$	$\surd$	$\surd$
(2) $L2a$	$\times$	$\surd$	$\surd$
G': sequential	Player 1	second-mover	Player 3
(3) $L2b$	$\surd$	$\surd$	$\surd$
(4) $L2a$	$\times$	$\surd$	$\surd$

Note: $\surd$ denotes choosing the (iterated, conditionally) dominant strategy at this position, $\times$ otherwise.

Table 1: Separation of $L2b$ and $L2a$

The games used in this experiment include 4 players, which separates $L2b$ and $L2a$, as well

[1] An appropriate solution concept for this sequential game is Iterated Conditional Dominance. Although I do not specify it every time, a dominant strategy in the sequential ring game is a "conditionally" dominant strategy. The connection between rounds of Iterated Conditional Dominance and steps of reasoning will be discussed in more detail in Section 3.3.

as $L3b$ and $L3a$. A total of 184 subjects participated in the experiment, and enough observations were collected from 179 of them to perform the analysis. There are 50 and 39 subjects classified as $L2$ or $L3$ respectively in the simultaneous games. More than half of these subjects failed to perform the extra step in the sequential game, suggesting that a considerable number of subjects are bounded by their ability.

I then perform a subject-by-subject type classification (Figure 2). Out of the 50 $L2$ subjects, 21 are confirmed to be $L2b$, who exhibit the ability to reason at least 3 steps in the sequential games, and 21 appear to be $L2a$, who could not finish the extra step (the remaining 8 subjects are classified as $L1$, $L3$ or unidentified). Out of the 39 $L3$ subjects, 20 are classified as $L3b$ and 15 as $L3a$ (the remaining 4 subjects are classified as $L0$ or unidentified). The results show that the subjects' belief and ability levels are mostly consistent throughout the experiment, and that about half of the subjects using two or three steps have reached their upper boundaries of reasoning.

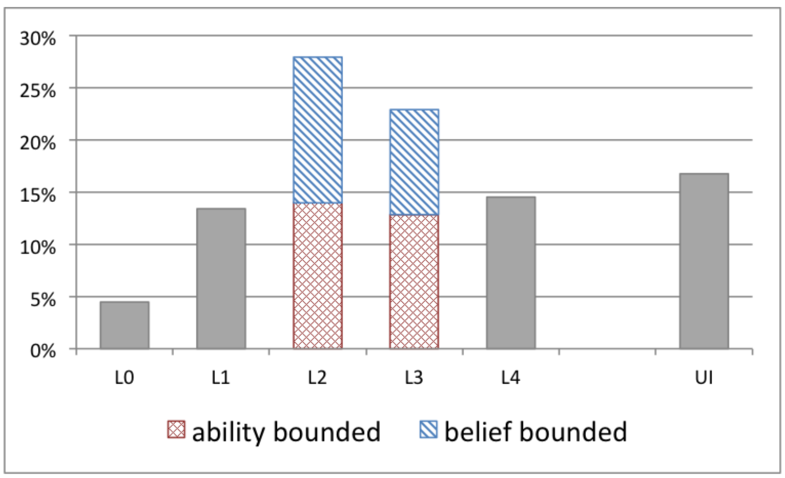


Figure 2: Decomposition of $L2$ and $L3$ subjects

To help better understand the correlation between cognitive ability and observed levels, the Cognitive Reflection Test (CRT) scores were collected from the participants after the main part of the experiment. The CRT score is a measure of cognitive ability in decision making. I find that the CRT scores increase with levels among the lower types, and then flatten out. Especially, I find that $L2b$ subjects performed much better in CRT than $L2a$. But among the players who use at least three steps of reasoning in the ring games, the differences in CRT scores are not significant.

Overall, I find the existence of both the high ability subjects responding to lower-order belief and the low ability subjects who could only think two or three steps, which shows large heterogeneity in subjects' reasoning ability. The results suggest that both subjects' belief and their reasoning ability could be the decisive factors of their exhibited levels. The CRT scores support the separation of $L2b$ and $L2a$ subjects.

The remainder of the paper proceeds as follows. The next section summarizes related literature. The theory, experimental design, and identification strategies are presented in Section 3 and Section 4. Section 5 is the experimental procedure. Section 6 reports the experimental results. Section 7 concludes.

2 Related Literature

The recent two decades saw the emergence of a vast literature on the level-k model. This section summarizes only the studies that are closest to this paper. The simultaneous ring games used in this study are similar to the ones proposed by Kneeland (2015). Her methodology and other studies on ring games will be discussed in detail in the following sections, and hence are not included here. There is also a class of literature assuming that Lk players believe that the opponents are drawn from a distribution of all the lower types, e.g. the Cognitive Hierarchy model from Camerer et al. (2004). Since the games in this study do not distinguish between the level-k model and the Cognitive Hierarchy model, players in this study are assumed here to have degenerate belief, as in the original level-k model.

There are a few studies looking at subjects' reaction to the information on the opponents' types. Given the information that the opponents are higher types, the high ability Lkb subjects should be able to respond by raising their own levels, while the ability-bounded Lka ones are not able to adjust their behavior. Some studies find positive results, suggesting that the participants are belief-bounded. For example, Palacios-Huerta and Volji (2009) invite chess players, presumably the higher types, and college students to play the centipede game, and find that they both exhibit higher levels when playing against chess players than against students. Agranov et al. (2012) find that in a 2/3 beauty-contest game, on average, subjects exhibit higher sophistication levels as the number of experienced players, some graduate students of Economics, increases in the group. In Slonim (2005), experienced subjects are found to respond to the experience levels of their opponents. Alaoui and Panta (2015) also find that subjects respond to the manipulation of their beliefs.

In some other studies, however, the results are mixed. For example, in Gill and Prowse (2015), who use repeated beauty contest games, the higher cognitive ability subjects respond positively to the cognitive abilities of their opponents, while the lower cognitive ability subjects do not. In addition, Georganas et al. (2015) find similar effects in their undercutting games, but not in the two-player guessing games.

A more direct way to identify whether subjects are best responding to their belief is to elicit their belief. Belief elicitation in strategic games has been studied in numerous papers. However, evidence is mixed on whether beliefs could be successfully elicited without altering behavior and whether subjects do act according to their stated beliefs. Some studies find that subjects' actual play in the games is not affected after belief elicitation (Nyarko and Schotter (2002), Costa-Gomes and Weizsacker (2008), Manski and Neri (2013)), while both Ruststrom and Wilcox (2009) and Gachter and Renner (2010) find that incentivized elicitation alters choices. In both Nyarko and Schotter (2002) and Manski and Neri (2013), most subjects appear to be best responding to their first-order belief. However, in Bhatt and Camerer (2005), only 66% of the choices match with the stated first-order belief. Costa-Gomes and Weizsacker (2008) also find that most choices are one step below the stated first-order belief. One possible explanation of the occasional failure of belief elicitation could be the hedging behavior of risk-averse subjects, as discussed in Kadane and Winkler (1988), Blanco et al. (2010) and Armantier and Treich (2013).

Additional strategies are developed to infer beliefs from observed levels. Arad and Rubinstein (2012) use a very simple undercutting game, but still find that subjects mostly use no more than three steps of reasoning. They conclude that this could not be due to obstacles in thinking, and thus must be due to non-equilibrium beliefs. Other methods include tracking players' information search (Costa-Gomes et al. (2001), Costa-Gomes and Crawford (2006) and Brocas et al. (2014)), or translating their beliefs from communication records between players (Burchardi and Penczynski, 2014). Bhatt and Camerer (2005) use brain image to search for the connection between the brain activities of making choices and stating belief.

The mixed findings in the previous studies, most of which are on aggregate levels, suggest heterogeneity in the population. Thus, this study seeks to identify players' belief and cognitive ability bounds at the individual level. However, the nature of some existing techniques, such as belief manipulation, makes it very hard to implement a within-subjects design, while belief-elicitation might not provide the right incentive when subjects are risk averse. This paper employs a novel identification strategy that only requires choice data, which circumvents the challenge from belief manipulation or belief elicitation and could easily achieve a within-subjects design.

Friedenberg et al. (2017) also explore the belief and reasoning in a ring game setting. However, their research question is fundamentally different from mine. Friedenberg et al. (2017) study the difference between two types of high-order beliefs, while this paper studies the relationship between belief and ability. Friedenberg et al.'s (2017) "cognitive bound" means how many reasoning steps the player forms theory of, but the theories might not be rational. For example, a player could believe that the opponent is using an irrational decision rule and thus best respond to it. Therefore, the orders of belief in the opponent's rationality might be lower than the orders of belief in whether the opponent has a decision rule, or belief in "cognition". Friedenberg et al. (2017) identify the gaps between beliefs in rationality and belief in cognition. Note that, in their study, a player's "cognitive bound" still depends on his belief of the opponent's cognitive bound. If the player believes that the opponent forms m levels of theories of play, then he would reason $m + 1$ steps. In this study, a player with an ability bound of k is capable of best responding through k steps of iterated reasoning, and the player's "ability bound" is independent of his belief. This study looks into the gap between the player's belief and ability. Friedenberg et al. (2017) find that 47% of their subjects have lower rationality bound than cognitive bound. Since cognitive bound must be lower than ability bound, the finding in this study is consistent with theirs.

Besides which, this paper finds positive correlation between CRT scores and observed levels, which extends the literature that explores the relationship between cognitive ability and strategic thinking. One of the closely related studies is Gill and Prowse (2015), who use the Raven's test as a measure of cognitive ability and find that the high ability subjects converge to equilibrium more quickly and earn more. Another related study, Georganas et al. (2015), uses four tests, including the CRT. They find that none of the test scores is a good prediction of observed levels, but CRT scores could predict earnings. In addition, Rydval et al. (2009) find that higher working memory, need for cognition, and premeditation are associated with a higher likelihood of obeying dominance. Works on the Beauty Contest Games also show that the subjects who score high in the CRT, or other tests of cognitive ability, play the strategies closer to equilibrium (Burnham et al., 2009; Schnusenberg and Gallo, 2011; Branas-Garza et al., 2012).

3 The Ring Games and the Cognitive Types

This section first presents the games used in the experiment, followed by the definitions of cognitive types in simultaneous games and sequential games.

3.1 The Games

The main part of the experiment consists of three sets of 4-player ring games, which are called SIMUL (the simultaneous games), SEQ-P2 and SEQ-P3 (the sequential games, see Figure 3). The SIMUL set contains two ring games, G1 and G2, with simultaneous moves. The two sets of sequential games are labeled by second-movers' positions. In the games of SEQ-P2, G3 and G4, Player 2 is the second-mover. Players 1, 3 and 4 move simultaneously in the first stage and Player 2 moves in the second stage after observing their actions. In the SEQ-P3 games, G5 and G6, Player 3 moves in the second stage. The sequence of moves and the information structure are common knowledge among all the players.

Each player has three actions to choose from, and the payoffs are represented by 3×3 matrices. Player 1's (P1) payoff is determined jointly by his own action and Player 2's (P2) action. Player 2's payoff is determined by his and Player 3's (P3) actions. Player 3's payoff depends on his and Player 4's (P4) actions. Player 4's payoff depends on his and Player 1's actions.

In these ring games, Player 4 always has strictly dominant strategies. Therefore in each game there is a unique strategy profile that survives iterated elimination of strictly dominated strategies in simultaneous games, or iterated conditional dominance in sequential games.

In addition, in each pair of rings (G1 and G2, G3 and G4, G5 and G6), the payoff matrices of the two Players 1, 2 and 3 positions are identical. The two Player 4 positions have the same three strategies, but the strategies are labeled in different orders. This feature, combined with the dominant strategies at Player 4 positions, is designed specifically for type classification, which will be discussed in the subsequent subsections.

3.2 Cognitive Types in Simultaneous Ring Games

In this subsection, I propose a model in which a player's type could be characterized by both his cognitive ability and his belief about the opponents' types. The closest setup is in Alaoui and Panta (2016). Similar assumptions could be found in Choi (2012), Strzalecki (2014) and Georganas et al. (2015).

3.2.1 Reasoning Steps

A player using one step of reasoning considers only his own situation, and lacks the strategic thinking that other players are also reasoning and best responding. In the ring games, such a player focuses on his own payoff matrix. He will be able to identify the dominated strategies if he has one, but will not pay attention to the dominated strategies in others' matrices.

To perform one more step of reasoning, the player needs to start developing one level of "Theory of Mind", or, as illustrated by Alaoui and Panta (2016), "one round of inspection". For example, if a player is doing the second step of reasoning in the simultaneous ring games, he would realize that, to find a best response, he should try to determine the action of the opponent to his right. He would then put himself in the right opponent's shoes and reason about the opponent's reasoning. The next step would be to develop a theory about the opponent's opponent's reasoning, and so on.

Players are assumed to be cognitively constrained. In a certain game, the maximum number of reasoning steps player i could perform, or his ability bound, is denoted by a_i. a_i reflects the player's cognitive ability. As suggested by Alaoui and Panta (2016), a_i might depend on the incentives of the game. But since the incentive level is not varied in this experiment, I do not consider cost-benefit analysis, and use a_i alone to capture the player's cognitive cost.

3.2.2 Beliefs and Cognitive Types

In a simultaneous ring game, Player i's cognitive type $t_i \in T_i$ is characterized by (a_i, b_i), where $a_i \in \{0, 1, 2, ...\}$ is the ability bound, and $b_i \in \Delta(T_{-i})$ represents the player's belief about his opponents' types. For simplicity, a player is assumed to hold the same belief about his 3 opponents. This study only considers degenerate beliefs, which restricts that $b_i \in T_{-i}$.

A player is allowed to hold the belief that the opponents reason more steps than his own ability bound a_i. In this case, a player is not able to form a concrete idea of the actions the opponents might take, due to his cognitive limit. Rather, he has an abstract belief of the sophistication

Player 1 \ Player 2	$a2$	$b2$	$c2$
$a1$	30	6	20
$b1$	4	20	32
$c1$	12	40	6

P1's Payoff

Player 2 \ Player 3	$a3$	$b3$	$c3$
$a2$	32	24	4
$b2$	36	20	10
$c2$	26	18	16

P2's Payoff

Player 3 \ Player 4	$a4$	$b4$	$c4$
$a3$	18	20	26
$b3$	14	8	36
$c3$	6	24	28

P3's Payoff

Player 4 \ Player 1	$a1$	$b1$	$c1$
$a4$	8	12	24
$b4$	20	16	32
$c4$	18	14	20

P4's Payoff

SIMUL: G1

Player 1 \ Player 2	$a2$	$b2$	$c2$
$a1$	30	6	20
$b1$	4	20	32
$c1$	12	40	6

P1's Payoff

Player 2 \ Player 3	$a3$	$b3$	$c3$
$a2$	32	24	4
$b2$	36	20	10
$c2$	26	18	16

P2's Payoff

Player 3 \ Player 4	$a4$	$b4$	$c4$
$a3$	18	20	26
$b3$	14	8	36
$c3$	6	24	28

P3's Payoff

Player 4 \ Player 1	$a1$	$b1$	$c1$
$a4$	18	14	20
$b4$	8	12	24
$c4$	20	16	32

P4's Payoff

SIMUL: G2

Player 1 \ Player 2*	$a2$	$b2$	$c2$
$a1$	16	32	8
$b1$	28	16	14
$c1$	14	10	32

P1's Payoff

Player 2* \ Player 3	$a3$	$b3$	$c3$
$a2$	6	30	28
$b2$	22	28	8
$c2$	4	20	36

P2*'s Payoff

Player 3 \ Player 4	$a4$	$b4$	$c4$
$a3$	22	8	24
$b3$	28	12	20
$c3$	32	4	18

P3's Payoff

Player 4 \ Player 1	$a1$	$b1$	$c1$
$a4$	24	20	32
$b4$	16	12	28
$c4$	22	18	8

P4's Payoff

SEQ-P2: G3

Player 1 \ Player 2*	$a2$	$b2$	$c2$
$a1$	16	32	8
$b1$	28	16	14
$c1$	14	10	32

P1's Payoff

Player 2* \ Player 3	$a3$	$b3$	$c3$
$a2$	6	30	28
$b2$	22	28	8
$c2$	4	20	36

P2*'s Payoff

Player 3 \ Player 4	$a4$	$b4$	$c4$
$a3$	22	8	24
$b3$	28	12	20
$c3$	32	4	18

P3's Payoff

Player 4 \ Player 1	$a1$	$b1$	$c1$
$a4$	16	12	28
$b4$	22	18	8
$c4$	24	20	32

P4's Payoff

SEQ-P2: G4

Player 1 \ Player 2	$a2$	$b2$	$c2$
$a1$	18	30	10
$b1$	36	8	14
$c1$	20	14	34

P1's Payoff

Player 2 \ Player 3*	$a3$	$b3$	$c3$
$a2$	20	10	26
$b2$	34	8	14
$c2$	16	32	10

P2's Payoff

Player 3* \ Player 4	$a4$	$b4$	$c4$
$a3$	8	18	36
$b3$	28	32	6
$c3$	16	40	8

P3*'s Payoff

Player 4 \ Player 1	$a1$	$b1$	$c1$
$a4$	8	24	16
$b4$	24	28	40
$c4$	18	6	32

P4's Payoff

SEQ-P3: G5

Player 1 \ Player 2	$a2$	$b2$	$c2$
$a1$	18	30	10
$b1$	36	8	14
$c1$	20	14	34

P1's Payoff

Player 2 \ Player 3*	$a3$	$b3$	$c3$
$a2$	20	10	26
$b2$	34	8	14
$c2$	16	32	10

P2's Payoff

Player 3* \ Player 4	$a4$	$b4$	$c4$
$a3$	8	18	36
$b3$	28	32	6
$c3$	16	40	8

P3*'s Payoff

Player 4 \ Player 1	$a1$	$b1$	$c1$
$a4$	18	6	32
$b4$	8	24	16
$c4$	24	28	40

P4's Payoff

SEQ-P3: G6

Note: * denotes second-movers.

Figure 3: The ring games

level of the opponents. Although such situations occur frequently in everyday lives, there are few theories on people's strategy against obviously stronger opponents. It is assumed that when a player faces a more sophisticated opponent, he would perform the maximum reasoning steps as he could. That is, he would reason up to a_i steps. The same assumption is used in Alaoui and Panta (2016).

This study does not attempt to separate all the types in this framework. Instead, the experimental design allows us to observe some mutually exclusive subsets of the type space, which are defined according to the classifications of Lk types.

Within this framework, $L0$ can be characterized by the set

$$T_i^0 = \{(a_i, b_i) | a_i = 0, b_i \in T_{-i}\}.$$

This type does not engage in any optimization[2]. $L0$ type is rarely observed in the real world and serves mainly as the anchoring point of analysis.

An $L1$ player performs one step of reasoning, that he does the optimization as if others do not think rationally. Being $L1$ actually implies the following cases

$$T_i^1 = T_i^{1b} \cup T_i^{1a} = \{(a_i, b_i) | a_i > 1, b_i \in T_{-i}^0\} \cup \{(a_i, b_i) | a_i = 1, b_i \in T_{-i}\}.$$

T_i^{1b} players ($L1b$) have higher cognitive ability than thinking one step. Nevertheless, they hold the belief that the opponents belong to the set T_{-i}^0, so their best response is to not reason about others' reasoning. A T_i^{1b} player's behavior is determined by his belief.

T_i^{1a} players ($L1a$), however, are cognitively constrained to think only one step. Although their belief might range over the whole type space, it is the ability level that determines the behavior.

In the level-k framework, $L2$ players behave as if they best respond to the $L1$s. Again, they can be decomposed to two cases:

$$T_i^2 = T_i^{2b} \cup T_i^{2a} = \{(a_i, b_i) | a_i > 2, b_i \in T_{-i}^1\} \cup \{(a_i, b_i) | a_i = 2, b_i \in T_{-i} \backslash T_{-i}^0\}$$

There is the T_i^{2b} type ($L2b$) who has high reasoning ability, but believes that others belong to $L1$ (T_{-i}^1), as well as the T_i^{2a} ($L2a$) type who is constrained by two steps of reasoning. T_i^{2a}'s belief ranges from T_{-i}^1 to all the higher types (excluding T_{-i}^0), but the cognitive bound stops him from best responding to the beliefs higher than T_{-i}^1.

Similarly, each Lk ($k \geq 2$), defined as best responding to $L(k-1)$, can be broken down to the belief-bounded T_i^{kb} type and the ability-bounded T_i^{ka} type:

$$T_i^k = T_i^{kb} \cup T_i^{ka} = \{(a_i, b_i) | a_i > k, b_i \in T_{-i}^{k-1}\} \cup \{(a_i, b_i) | a_i = k, b_i \in T_{-i} \backslash (\cup_{m=0}^{k-2} T_{-i}^m)\},$$

[2] Usually it is assumed that $L0$ does not form any belief at all ($b_i \in \emptyset$). Here I use $b_i \in T_{-i}$ for $L0$ so that other types could be defined based on T_i^0.

with T_i^{kb} and T_i^{ka} corresponding to the Lkb and Lka types respectively.

3.3 Cognitive Types in Sequential Ring Games

In the sequential ring games, a player i's type is characterized by $t'_i = (a'_i, a''_i, b'_i) \in T'_i$, where $a'_i \in \{0, 1, 2, ...\}$ and $a''_i \in \{0, 1\}$ are the player's the ability bounds as first-movers and second-movers respectively, and $b'_i \in T'_{-i}$ represents the player's belief about his opponents' types[3].

The relationship between the reasoning steps in sequential games and sequential rationality, or rounds of iterated conditional dominance (ICD) (Battigalli and Siniscalchi, 2002; Shimoji and Watson, 1998), is worth mentioning. Unlike in the simultaneous games, the steps of reasoning do not coincide with the rounds of ICD in the sequential ring games. For example, in the 4-player sequential game of SEQ-P3, Player 2 will be able to identify the iterated dominated strategies in his own payoff matrix by performing only two rounds of ICD[4]. However, this implies that he understands that Player 3 is best responding, and that Player 4 is also acting optimally, which requires that he develops 3 levels of reasoning.

In addition, although I do not use the terms of sequential rationality, such as kth-order strong belief in rationality, b'_i is capturing player i's belief about the opponents who are active in different information sets.

4 Identification Strategy

4.1 Opponent Systems in Ring Games

Before presenting the identification strategies, it is useful to discuss the opponent system in ring games. In a simultaneous or sequential ring game, define the **first direct opponent** of a player to be the opponent whose action would affect his payoff. Therefore, the first direct opponent of Player i would be Player $(i + 1)$ for $t = 1, 2, 3$ and would be Player 1 for $i = 4$. Then the **kth direct opponent** of a player could be defined to be the opponent whose action would affect his $(k - 1)$th direct opponent's payoff. In ring games, the set of kth direct opponent of a player is a singleton for all k.

In addition, define the **kth-order opponents** of player i as the opponents about whom player i forms kth-order belief. In the simultaneous games, a player's kth-order opponent is his kth direct opponent, for each player only needs to form belief about the opponent who affects his

[3]Note that even Player i is a first-mover position, an a''_i is still assigned as a character to this player, and vice versa for the second-mover position. Therefore such a type characterization could be used on all player positions, which makes it more convenient to define higher-order beliefs.

[4]In the first round of ICD, Player 2 would eliminate the conditionally dominated strategies in all the information sets of the game, which means that he identifies Player 4's conditionally dominated strategies and Player 3's (second-mover) conditionally dominated strategies in the three information sets where Player 3 is active. In the second round, with the reduced game, Player 2 would identify the conditionally dominated strategies in his own matrix.

payoff.

In a sequential ring game, however, in order to derive the action of a second-mover, a player needs to form a theory not only about the second-mover, but also the second-mover's first direct opponent. The table below uses the games in SEQ-P2 as an illustration. Player 2 is the second-mover. Therefore, in order to best respond to Player 2, Player 1 needs to form 1st-order belief about both Players 2 and 3, so they are both Player 1's 1st-order opponents. But only Player 2 is Player 1's first direct opponent.

Player 1	Player 2 (second-mover)	Player 3	Player 4
	P1's 1st direct opponent	P1's 2nd direct opponent	P1's 3rd direct opponent
	P1's 1st-order opponent	P1's 1st-order opponent	P1's 2nd-order opponent

Table 2: Demonstration of Opponent Systems in SEQ-P2

4.2 Identify Reasoning Steps in Simultaneous Ring Games

Subjects' choices in the simultaneous ring games could be used to infer their reasoning steps. That is, each subject could be put in to one of the T_i^k sets, where $k = 0, 1, 2, 3, 4$ represents the steps of reasoning.

A player using k steps of thinking will reason through his own problem, his first direct opponent's problem, his second direct opponent's problem, and so on. The kth step would be the $(k-1)$th direct opponent's decision problem. He will stop here and will not examine the kth direct opponent's strategies. This leads to the first identifying assumption:

A1: A player using k steps of reasoning does not respond to the changes in mth direct opponent's payoff, for all $m \geq k$.

$A1$ is developed from the **exclusion restriction** (ER) in Kneeland (2015) and is modified so that it could be applied to both simultaneous and sequential ring games[5]. $A1$ implies that a player's choice will not be affected by the changes in a distant opponent's decision situation, if he is not reasoning all the way to that opponent.

In the simultaneous ring games, $A1$ is used to identify subjects' reasoning steps. In other words, each subject could be put into a Lk type, or a T_i^k set, though it still remains unclear whether the subject's reasoning step is determined by his belief (Lkb or the T_i^{kb} set) and that he actually has higher reasoning ability, or it is determined by his ability bound (Lka or the T_i^{ka}

[5]The assumption ER in Kneeland (2015) says that a player with kth- but not higher-order belief in rationality does not respond to the changes in $(k+1)$th- or higher-order opponents' payoffs.

set). Separation of Lkb and Lka subjects requires additional information from the sequential ring games.

The identification strategy works as follows. In G1 and G2, the payoff matrices are identical in the Players 1, 2 and 3 positions respectively. The only difference is in the payoff matrices of Player 4. Hence $A1$ implies that an $L1$ type, who reasons one step in the simultaneous games, chooses the dominant strategies as Player 4 but makes the same choice in both rings as Players 1, 2 and 3. The behavior pattern of other Lk types could also be predicted. An $L2$ subject makes the same choices as Players 1 and 2 in both G1 and G2, and chooses the (iterated) dominant strategies as Players 3 and 4. An $L3$ subject makes the same choice at the two Player 1 positions and chooses the (iterated) dominant strategies as Players 2, 3 and 4. $L4$ or higher types choose the (iterated) dominant strategies at all player positions.

It should be noted that both $A1$ and ER from Kneeland (2016) imply that a player always chooses the same action when presented with the same strategic decision problem. If this assumption does not hold, it could cause large noise in type classification. See Lim and Xiong (2016) and Jin (2016) for detailed discussions.

Another identification strategy commonly used in the literature is to assume that $L0$ chooses all the possible actions with equal probability, and solves higher types' best responses recursively. The predicted action profiles of this assumption are special cases of that of $A1$. The type classification using the uniformly randomizing $L0$ assumption appears to be less robust among lower types, but the pattern in the main result holds for higher types. The more detailed analysis and results will be reported in Appendix C.2.

4.3 Identify Belief-bounded and Ability-bounded Subjects

4.3.1 Assumptions on the Similarity of Simultaneous and Sequential Games

The sequential ring games are framed in such a way that the path of reasoning stays the same as in the simultaneous games. More specifically, no matter whether there is a second-mover, the reasoning path always naturally follows as the players' own problem, the first direct opponent's problem, the second direct opponent's problem, and the third direct opponent's problem. The identification strategy of this study relies on this similarity, so that meaningful comparisons could be made between subjects' behavior in simultaneous and sequential games.

To make it clearer, I state three assumptions on the relationship between a subject's types in the simultaneous and sequential ring games.

A2: As a first-mover, a subject's upper bound of reasoning steps stays the same in simultaneous and sequential ring games. That is, $a_i = a_i'$, $\forall i$.

As a first-mover, one needs to go through the same thought process to develop theories' of play and theories of other players' play in both the simultaneous and sequential games. It is assumed in $A2$ that a subject's ability level should not change as a first-mover.

Due to the same argument, a subject's belief in other first-movers' rationality should not change either.

A3: A subject's belief hierarchy about their opponents' reasoning ability as first-movers stays the same in simultaneous and sequential ring games.

Suppose the 1st-order belief of player i can be written as $b_i^1 = b_i = (\hat{a}_{-i}^1, \hat{b}_{-i}^2)$ in simultaneous games, and $b_i'^1 = b_i' = (\hat{a'}_{-i}^1, \hat{a''}_{-i}^1, \hat{b'}_{-i}^2)$ in sequential games.

Let O_i^k denote the set of the *kth-order opponents* of Player i. The kth-order beliefs of player i can be written as $\hat{b}_{O_i^{k-1}}^k = (\hat{a}_{O_i^k}^k, \hat{b}_{O_i^k}^{k+1})$ and $\hat{b'}_{O_i^{k-1}}^k = (\hat{a'}_{O_i^k}^k, \hat{a''}_{O_i^k}^k, \hat{b'}_{O_i^k}^{k+1})$. The assumption implies that $\hat{a}_{O_i^k}^k = \hat{a'}_{O_i^k}^k$, $\forall k$ and i.

As for the second-movers in sequential games, although it also takes one step of reasoning to best respond, the task complexity is not exactly the same as the one step reasoning in the simultaneous game. Thus, I only assume that the task for the second-mover is easier, if not the same, than the task for one reasoning step in the simultaneous games.

A4: If a subject's kth-order belief says that the opponents reason at least one step in the simultaneous games, then in his kth-order belief the opponents are capable of best responding as the second-movers in the sequential games. That is, if $\hat{a}_{O_i^k}^k \geq 1$, then it must be that $\hat{a''}_{O_i^k}^k = 1$, $\forall k$ and i.

There are some concerns that reasoning in sequential games is easier than in simultaneous games. For example, if a player position's first direct opponent is the second-mover, it is more likely to induce the subject to start examining his opponent's situation. In this regard, the ability bound might be higher in sequential games for some subjects. If this is true, then it implies that this study is identifying an upper bound of the proportion of the subjects whose ability bounds exceed belief bounds. It will be discussed in more detail in Section 4.3.2. The tests on whether ability bounds stay the same across different games will be presented in the results part.

4.3.2 Identification Strategy

With these three assumptions, it could be predicted how many steps a belief-bounded or ability-bounded player would reason in the sequential games. In addition, if a player does not reason enough steps so that he could identify the iterated dominant strategy at a certain position, by assumption $A1$, he chooses the same action at this position in that pair of games (the SEQ-P2 pair or the SEQ-P3 pair). In this way, the behavioral patterns of all the Lkb and Lka types could be predicted, as shown in Table 3.

$L2b$ and $L2a$ could be separated using the Player 2 positions of SEQ-P3. An $L2b$ subject belongs to the set $T_i^{2b} = \{(a_i, b_i) | a_i > 2, b_i \in T_{-i}^1\}$. Therefore, in sequential games he holds the

		P1	P2	P3	P4			P1	P2	P3	P4
$L1b$	SEQ-P2	$\times$	$2nd$	$\times$	$\surd$	$L1a$	SEQ-P2	$\times$	$2nd$	$\times$	$\surd$
	SEQ-P3	$\times$	$\times$	$2nd$	$\surd$		SEQ-P3	$\times$	$\times$	$2nd$	$\surd$
$L2b$	SEQ-P2	$\times$	$2nd$	$\surd$	$\surd$	$L2a$	SEQ-P2	$\times$	$2nd$	$\surd$	$\surd$
	SEQ-P3	$\times$	$\surd$	$2nd$	$\surd$		SEQ-P3	$\times$	$\times$	$2nd$	$\surd$
$L3b$	SEQ-P2	$\surd$	$2nd$	$\surd$	$\surd$	$L3a$	SEQ-P2	$\times$	$2nd$	$\surd$	$\surd$
	SEQ-P3	$\surd^*$	$\surd$	$2nd$	$\surd$		SEQ-P3	$\times$	$\surd$	$2nd$	$\surd$
$L4b$	SEQ-P2	$\surd$	$2nd$	$\surd$	$\surd$	$L4a$	SEQ-P2	$\surd$	$2nd$	$\surd$	$\surd$
	SEQ-P3	$\surd$	$\surd$	$2nd$	$\surd$		SEQ-P3	$\surd$	$\surd$	$2nd$	$\surd$

Note: $\surd$ denotes choosing the (iterated conditional) dominant strategy at this position, $\times$ otherwise. If a player does not identify the (iterated conditional) dominant strategy at a certain position (e.g. P1 of SEQ-P2), then according to assumption "$A1$", he chooses the same action at this position in the two games (e.g. he chooses the same action at P1 in the two games of SEQ-P2).
"2nd" denotes second-movers. All the types but $L0$ should play the dominant strategies in the subgame as second-movers.
(*): $L3b$ plays the iterated conditional dominant strategy only if he believes that the opponents are $L2b$

Table 3: Predicted action profiles of each Lkb and Lka type

belief that the first-mover opponents reason one step (either because $\hat{a'}^1_{-i} = 1$ or because $\hat{a'}^1_{-i} > 1$ and $\hat{a'}^2_{O_i^2} = 0$). At the Player 2 position, he would believe that Player 4 chooses the dominant strategy. In addition, given $A4$, he should also believe that Player 3 best responds after observing Player 4's action ($\hat{a''}^1_{-i} = 1$). Hence, an $L2b$ subject will be able to solve for his own best response through 3 steps of reasoning.

An $L2a$ subject, who belongs to the set $T_i^{2a} = \{(a_i, b_i)|a_i = 2, b_i \in T_{-i} \backslash T_{-i}^0\}$, is only able to think two steps and is thus not able to best respond as P2 of SEQ-P3.

Similarly, $L3b$ and $L3a$ could be separated using the Player 1 position of SEQ-P2. An $L3b$ subject belongs to the set $T_i^{3b} = \{(a_i, b_i)|a_i > 3, b_i \in T_{-i}^2\}$, which implies that he believes that the first-mover opponents think two steps, and that the second-mover best responds. Thus, as Player 1, he would believe that Player 3 chooses the iterated conditional dominant strategy, and that the second-mover, Player 2, best responds to his observation of Player 3's action. Since $L3b$ could reason at least 4 steps, he is able to reason all the way back to his own problem and best respond.

An $L3a$ subject belongs to the set $T_i^{3a} = \{(a_i, b_i)|a_i = 3, b_i \in T_{-i} \backslash (T_{-i}^0 \cup T_{-i}^1)\}$. No matter what belief he holds, his ability bound restricts him from using more than 3 steps of reasoning, and thus will not be able to best respond as Player 1.

In addition, by comparing $L3$ subjects' behavior in SEQ-P3 and SEQ-P2, it could be tested whether $L3b$ subjects believe that their opponents are the belief-bounded $L2b$ or the ability-bounded $L2a$. In SEQ-P2, $L2b$ or $L2a$ are indistinguishable. A subject classified as $L3b$ might believe that the opponents are $L2b$ or $L2a$. But in SEQ-P3, Player 1 needs to believe that Player 2 is $L2b$ to play the best response. Therefore, if there exists any subject who believes that the opponents are $L2a$, he will best respond as P1 in SEQ-P2, but not as P1 in SEQ-P3.

With this design, $L1b$ and $L1a$ cannot be separated. $L4b$ and $L4a$ could be separated with 5-player ring games, which are not used in this experiment.

What if $A2$ does not hold well, and some subjects could reason more steps in sequential games, as discussed before? In this case, some Lka subjects might be classified as Lkb because they demonstrate higher reasoning ability in the sequential games. But those who are identified as Lka are guaranteed to be Lka. If so, as discussed before, this study provides an upper bound of the proportion of Lkb subjects.

5 Experimental Procedures

The experiment was conducted at the Experimental Social Science Laboratory (XLab) at UC Berkeley. A total of 184 subjects, who were recruited from Berkeley undergraduate classes, participated in 6 different sessions, with between 20 to 36 subjects in each session. Each experimental session lasted about one and a half hours. The average earnings were $25, plus a $5 participation fee, which were paid in private after each session.

The data was collected through an online interface. Subjects were not allowed to interact directly, and their identities were kept confidential. After subjects read the instructions, the instructions were read aloud by an experimenter. Then the subjects were given a short quiz to test their understanding of the game structure, followed by 4 unpaid practice games to help them get familiar with the interface.

In the main part of the experiment, the subjects played at the 24 positions of the games in a random order. In each game they were matched with a new group. Though the subjects were not allowed to write during the experiment, the online interface allowed them to mark any cell in the payoff matrices by a click of the mouse. In this way they were able to easily track the equilibrium strategies across matrices. The subjects were not allowed to make changes once they had confirmed their choices.

There was a time limit of 60 seconds for the subjects to complete each game. If they failed to choose in one game, the earnings for this game would be zero, and the system would randomly pick from the three choices for them to calculate the payoffs of their opponents. The second-movers were given an additional 30 seconds after the first-movers had submitted their choices[6].

[6]There are 12 subjects who failed to choose in one game (out of 184 $\times$ 24 games in total). For 7 out of the 12 subjects, missing one choice does not affect their type identification. Five subjects (406, 412, 610, 814, 1017) are affected and are excluded from the type distribution presented in the following section.

The decision time of each game was recorded for each subject.

After the subjects had completed all 24 games, they took the Cognitive Reflection Test. The CRT is composed of three short questions and is designed to measure subjects' cognitive ability (Frederick, 2005).

At the end of each session, subjects finished a survey of their demographic characters. One of the 24 games was randomly chosen for payment. One out of the three questions in the CRT were chosen for payoff and the subjects got $0.25 if their answers were correct.

6 Experimental Results

6.1 Data Description

The percentage of subjects who comply with iterated dominance at each of the 24 positions is reported in Table 4[7]. For the second-movers, I look at whether they choose the dominant strategies in the subgames.

		Player 1	Player $2^{(A)}$	Player $3^{(C)}$	Player 4
SIMUL	G1	34.2%	56.0%	77.7%	97.3%
	G2	46.7%	54.3%	79.3%	97.8%
		Player 1	Player 2 (2nd)	Player 3	Player 4
SEQ-P2	G3	47.3%	95.1%	80.4%	96.7%
	G4	42.9%	93.5%	77.7%	94.6%
		Player 1	Player $2^{(B)}$	Player 3 (2nd)	Player 4
SEQ-P3	G5	49.5%	71.2%	96.2%	98.4%
	G6	51.6%	65.2%	96.2%	97.3%

Note: $N = 179$.(2nd) denotes second stage movers.

Table 4: Compliance rates of (iterative) dominating strategies

The compliance rates are quite high at all of the Player 4 positions and the second-mover positions (Player 2 of SEQ-P2 and Player 3 of SEQ-P3). Over 95% of the subjects choose the dominant strategies at these positions, which suggests that the majority of the participants understand the payoff structure and are capable of identifying strict dominance. The compliance

[7] Out of the 184 participants, 172 finished all the choices within the time limit; 7 failed to choose in one game but the missing choice does not affect their type classification. The following analysis is based on the 179 subjects. The 5 subjects are excluded because they failed to choose in one game and the missing choice affects their type classification.

rates decrease as the required reasoning steps go up, which is consistent with the prediction of the level-k model: fewer players achieve higher levels of reasoning.

A trace of the treatment effects of the sequential moves could be found in the differences in compliance rates. For example, while the P2 positions of both SIMUL and SEQ-P3 require three steps of reasoning, it requires $L3$ belief in SIMUL (marked as "A"), but only $L2$ belief in SEQ-P3 (marked as "B"). If subjects' behavior is solely determined by their beliefs, we should expect higher compliance rates at P2 of SEQ-P3 ($B > A$), as more people have 2nd-order belief in rationality compared to 3rd-order belief. Such a pattern could indeed be found in the data. However, the compliance rates at P2 of SEQ-P3 are lower than P3 of SIMUL (marked as "C", $C > B$), which requires $L2$ belief but only two steps of reasoning. It might imply that not every subject responding to $L2$ belief in the simultaneous games is able to proceed to three steps of thinking. A similar pattern can be found for $L3$ subjects.

6.2 Observed Lk Behavior in Simultaneous Games

A subject would be identified as Lk in the simultaneous games if he holds at least Lk belief and has the ability to perform at least k steps of reasoning. Up to $L4$ behavior could be identified from subjects' choices in the simultaneous games.

The type classification method is from Kneeland (2015). It is assumed that each subject's behavior is determined by a single type, which remains constant throughout the experiment. A subject i deviates from his Lk type's choice profile with probability ϵ_{ik}, which is i.i.d. across games. When a subject deviates from his own type, it is assumed that he chooses the other two strategies with equal probability. The likelihood of a player i being type k given his action profile can be defined as

$$d_{ik}(\epsilon_{ik}, x_{ik}) = (1 - \epsilon_{ik})^{G - x_{ik}} (\frac{\epsilon_{ik}}{2})^{x_{ik}}, \tag{1}$$

where G is the number of games and x_{ik} is the number of observations that do not match the predicted profile of type k.

A subject is assigned to the type k with the highest likelihood d_{ik}, which is equivalent to finding the lowest number of deviations x_{ik}. If a subject's action profile matches exactly with a type's predicted profile, he will be assigned to this type. However, if $\min_k(x_{ik}) > 0$, there might be more than one minimum x_{ik}. Following Kneeland (2015), this subject will be assigned to the lowest type that has the minimum number of deviations.

If an action profile deviates too much from the predicted profiles of $L1$-$L4$, he would be labeled as the irrational $L0$ or the *unidentified*. The unidentified ones are defined as deviating from Lk predictions, but picking the dominant strategies as Player 4 in both two rings. Hence, they are at least capable of avoiding dominated strategies and should be distinguished from the irrational, unpredictable $L0$ type. The unidentified subjects might be using some rules of their own that cannot be captured by the level-k model.

Therefore a cutoff point is needed so that those subjects with $\min_k(x_{ik})$ larger than the cutoff will be assigned to $L0$ or the unidentified group. Table 5 reports the type assignment results with the cutoffs being 0, 1 or 2 deviations. In the first row, when a subject cannot be matched exactly to a Lk type, he is assigned to $L0$ or unidentifiable. This seems to be too strict as there are over 60% of the subjects left unidentified. When allowing for 1 deviation, the share of the unidentified subjects drops down to 15%, with more subjects being assigned to one of the four Lk types. If the 2-deviation cutoff is used, there is a further drop in the number of the unidentified subjects and an increase in $L1$ and $L2$. The numbers of higher types do not change.

Level	$L0$	$L1$	$L2$	$L3$	$L4$	UI
0 deviation	8	5	17	17	19	113
	4.5 %	2.8 %	9.5 %	9.5 %	10.6 %	63.1 %
random choice	89.0 %	0.4 %	0.1 %	0.1 %	0.0 %	10.4 %
1 deviation	7	26	50	39	30	27
	3.9 %	14.5 %	27.9 %	21.8 %	16.8 %	15.1 %
random choice	86.4 %	4.4 %	1.0 %	0.3 %	0.2 %	7.7 %
2 deviations	5	40	56	39	30	9
	2.8 %	22.3 %	31.3 %	21.8 %	16.8 %	5.0 %
random choice	72.5 %	18.9 %	4.2 %	1.5 %	0.7 %	2.2 %

Note: $N = 179$. Each subject is classified as $L1 - L4$ with no more than 0, 1 or 2 deviations from the predicted action profiles. Otherwise they are assigned to $L0$ or the unidentified group. The subjects classified as unidentified are able to choose dominant strategies as Player 4 but do not match any of the predicted patterns. They are at least rational, which makes them different from $L0$. The random choices are simulated with 10,000 randomly choosing subjects.

Table 5: Type assignment from SIMUL games

To determine which cutoff is the most appropriate in this study, a sample of 10,000 random choosing subjects is simulated and analyzed through the type assignment process. This analysis focuses on how many of these subjects could be correctly assigned to $L0$ and how many are wrongly assigned to one of the Lk types. When allowing for 1 deviation, over 86% of them are classified as $L0$, and around 5% go to the Lk types. It does not differ too much from the 0-deviation cutoff. However, when allowing for 2 deviations, over 25% of the random choosing subjects are assigned as Lk, which is too high to be acceptable. So it appears that the 1-deviation cutoff is the most appropriate, for it gives reliable results and provides enough observations of the Lk types for the following analysis. The results using the 0-deviation and 2-deviation cutoffs, from which a similar pattern could be found as in the main results, are reported in Appendix C.1.

The type classification in Kneeland (2015) also uses the 1-deviation cutoff. The 1-deviation cutoff type distribution in my data is very close to the distribution found in Kneeland (2015).

If the unidentified subjects are excluded, the Fisher's exact test comparing these two categorical distributions yields a p-value of 0.926, suggesting that they are statistically not different. However, the proportion of unidentified subjects, 15.1%, is much larger compared to the 1.2% in Kneeland's (2015) data[8], which is due to larger deviation rates of my subjects. It should be noted that even with the 2-deviation cutoff and thus only 5% unidentified subjects, Fisher's exact test still does not reject that the distribution in this study is different from Kneeland's (2015), though with a lower p-value of 0.853.

Just as the one in Kneeland (2015), this type distribution is relatively higher than the findings in the literature (for example, Stahl and Wilson (1995) and Costa-Gomes et al. (2001)), which might be attributed to the special features of ring games. There are 27.9% and 21.8% of the subjects classified as $L2$ and $L3$ respectively with the 1-deviation cutoff, which serves as the starting point of the following analysis on the separation of Lkb and Lka.

6.3 Are Observed Levels Determined by Belief or Ability?

In order to separate the belief-bounded and ability-bounded types, I classify subjects using the 20 first-mover choices from SIMUL and the two sets of sequential games (excluding the 4 choices as second-movers) with a likelihood function similar to (1). Six types ($L1$, $L2a$, $L2b$, $L3a$, $L3b$[9], $L4$) are included in the type assignment. The predicted actions of these types could be found in Section 4.3.

Subjects are assigned to a type with no more than 3 deviations from the type's predicted action profile[10]. When a subject deviates too much from a predicted profile, he is assigned to the unidentified category if he chooses 5 out of the 6 dominating strategies at Player 4 positions correctly, otherwise he is classified as $L0$. A randomly choosing subject has only 0.2% of a chance to correctly choose 5 dominant strategies.

Result 1 About half of the $L2$- and $L3$-behaving subjects in the simultaneous games are the belief-bounded Lkb types who have higher ability than the exhibited levels, the rest are Lka types who could perform, at most, two or three steps of reasoning.

Table 6 shows how the identified types change from using the 8 positions in the simultaneous games to using the 20 first-mover positions in the simultaneous plus sequential games. The

[8]Kneeland (2015) does not include a category of unidentified subjects, since there is only one such subject in her main treatment. This subject is assigned to $L0$ in her original paper.

[9]$L3b$ are assumed to hold the belief that the opponents are $L2b$. Here I do not include the type who believes that the opponents are $L2a$. More detailed discussion will be provided in the **Result 3** part in this section, which shows that subjects mostly believe that their opponents are belief-bounded rather than ability-bounded.

[10]The 3-deviation results are used in the main analysis, as the probabilities of subjects making 3 errors or fewer out of 20 decision tasks are roughly comparable to the probabilities of them making no or one error out of 8 decisions, if their error rates are between 10% and 20%. The results using 2-deviation or 4-deviation cutoffs are reported in Appendix A.

		Baseline + Sequential Types								
		$L0$	$L1$	$L2a$	$L2b$	$L3a$	$L3b$	$\geq L4$	UI	sum
Baseline Types	L0	**6**	0	0	0	0	0	0	1	7
	L1	0	**22**	1	1	0	1	0	1	26
	L2	0	3	**21**	**21**	0	1	0	4	50
	L3	1	0	0	0	**20**	**15**	0	3	39
	L4	0	0	0	1	1	0	**26**	2	30
	UI	1	1	1	1	0	1	0	**22**	27
	sum	8	26	23	24	21	18	26	33	179

Note: $N = 179$. Subjects are assigned to a type with no more than 3 deviations. Otherwise they are classified as $L0$ or unidentified. The subjects classified as unidentified are able to choose 5 out of 6 dominant strategies as player 4 but do not match any of the predicted patterns. The numbers in bold fonts represent the subjects who remain in the same Lk category in the two type classifications.

Table 6: Type assignment according to the observations from SIMUL, SEQ-P2 and SEQ-P3

numbers in bold fonts represent the subjects who remain in the same Lk category in the two type classifications. Most of the $L1$ and $L4$ subjects stay at the same levels when more observations are included, and all of the $L0$ subjects in the simultaneous games are still identified as $L0$. Overall, only approximately 15% of the subjects fall into a different category, and the majority of the subjects appear to be quite consistent across the three sets of games.

21 out of the 50 subjects who exhibit $L2$ behavior in the simultaneous games are identified as the ability-bounded $L2a$, who could do, at most, two steps of reasoning, compared with the other 21 $L2b$, who have higher ability and could proceed to three reasoning steps and still respond to their $L2$ belief. Of the 39 $L3$ subjects in the simultaneous games, more than half are identified as $L3a$, and 15 are identified as the higher ability $L3b$. The results suggest that around half of the $L2$ and $L3$ behavior observed in the simultaneous games is due to lack of ability to think further.

As previously mentioned, there are some concerns that subjects' belief bounds and ability bounds might not remain the same in simultaneous and sequential games, especially since sequential reasoning might be considered easier. How would such inconsistency affect the observed type distribution?

First, consider a subject who has higher belief bounds in the sequential ring games. If these belief bounds are below ability bounds, he would behave as different Lk types. For example, there is one subject who is classified as $L1$ in SIMUL, but $L2b$ if we look at all the games. His belief bound must have changed. Fortunately, such inconsistency would only be reflected in the switched levels, but would not affect the ratio of Lka and Lkb subjects. Therefore, if we exclude all the subjects who receive different type classifications, and focus only on the numbers in bold in Table 6, inconsistency in belief bounds should not affect the conclusion that half of the $L2$ and $L3$ subjects are ability bounded.

If some subjects have the same belief bounds, but higher ability bounds in the sequential games, the number of Lkb subjects might be overestimated. For example, a subject might have $L2$ belief, and an ability bound of 2 in simultaneous games and 3 in sequential games. He is actually ability-bounded in the simultaneous games by definition, but would be assigned to the belief-bounded $L2b$. In this case, the results here provide an upper bound of the proportion of the belief-bounded subjects who have higher abilities than the exhibited levels.

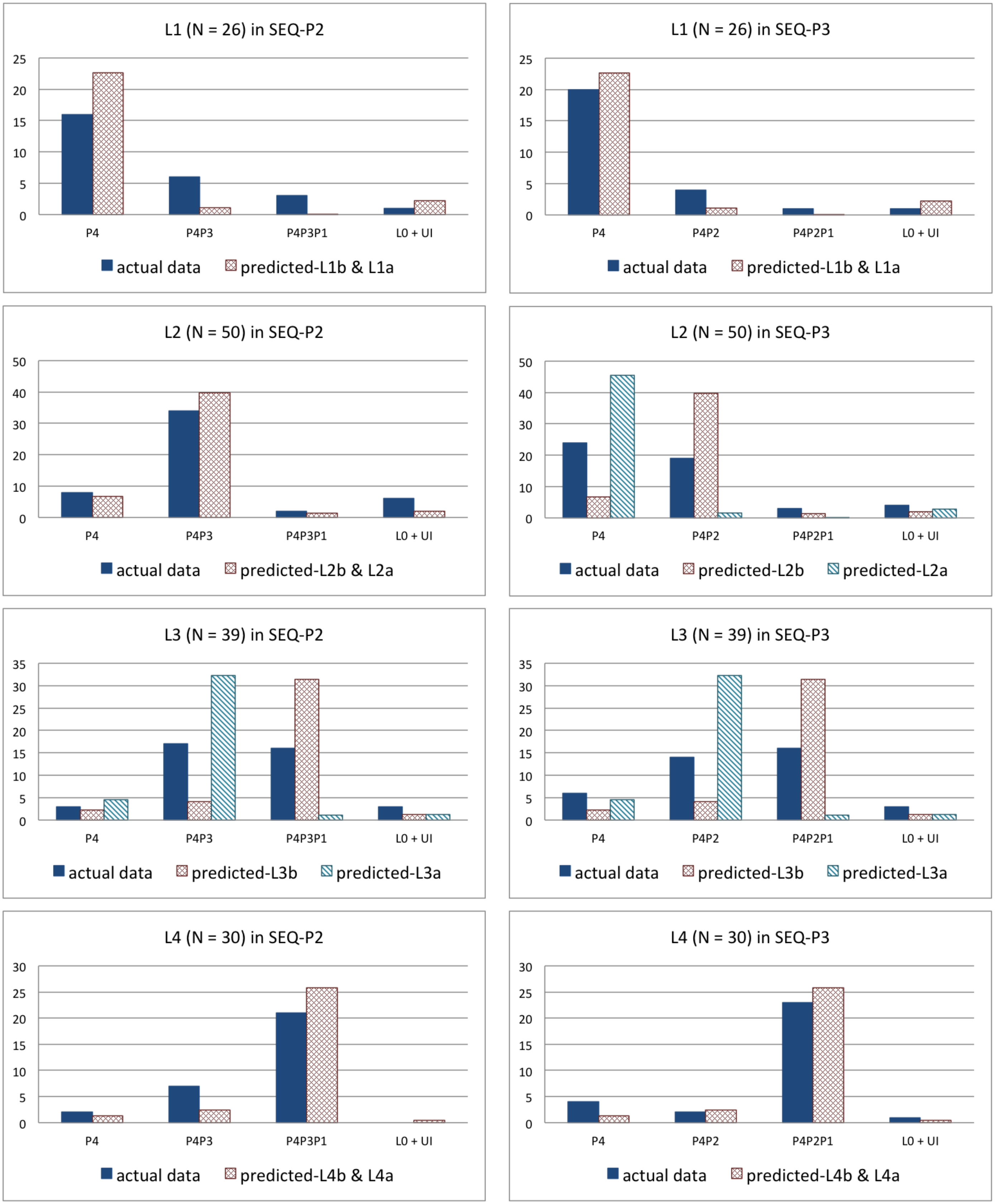
Nevertheless, there might not be a large amount of subjects with such increased ability bounds, for increased ability bounds could also lead to upward type switch and there are few such cases. For example, consider a subject who has $L2$ belief, and an ability bound of 1 in simultaneous games and 2 in sequential games. He would be classified as $L1$ and $L2a$ respectively. There appear to be very few cases of type switch from $L1$ to $L2a$, $L2$ to $L3a$ and $L3$ to $L4$. Thus, the proportion of Lkbs might have been overestimated, but the bias is not likely to be large.

I next look into the type switch patterns in more detail to analyze subjects' consistencies across the three sets of games.

Result 2 Subjects' behavioral patterns in the sequential games are close to the theoretical predictions assuming that their belief levels and ability levels are consistent across simultaneous and sequential games. Most subjects fall into the predicted categories.

Figure 4 gives the actual type switch patterns of the identified Lk type from SIMUL. In each diagram, the solid bar describes the behavior of these Lk types in SEQ-P2 or SEQ-P3. According to Table 3, subjects' behavior could be sorted into three categories based on their choices as first-movers in the sequential games. In SEQ-P2, the subjects could be playing the (iterated conditionally) dominant strategies only at P4 position, at P4 and P3 positions, or at all of the P4, P3 and P1 positions. In SEQ-P3, the three categories are playing the (iterated conditionally) dominant strategies at P4 position, at P4 and P2 positions, or at P4, P2 and P1 positions. The three categories correspond to the subjects best responding to $L1$, $L2$ and $L3$ belief in the sequential games. Each subject could be assigned to one of the categories by the method in Section 6.2 and the 1-deviation cutoff[11].

[11] According to $A1$, a subject best responding to $L1$ belief in SEQ-P2 plays dominant strategies as Player 4, but chooses the same actions as Players 1 and 3. This gives the predicted action profile for the "P4" category in SEQ-P2, which includes $L1b$ and $L1a$. Similarly, a subject responding to $L2$ belief in SEQ-P2 should play the (iterated conditionally) dominant strategies as Players 3 and 4 and choose the same action as Player 1, which gives the action profile for the "P4P3" category in SEQ-P2, including $L2a$, $L2b$ and $L3a$. A subject responding to $L3$ belief in SEQ-P2 should play the (iterated conditionally) dominant strategies as Players 1, 3 and 4, which is the "P4P3P1", including $L3b$, $L4a$ and $L4b$. The actions of the three categories in SEQ-P3 could be predicted in the same way.



Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 ("P4"), at P4 and P3 ("P4P3"), and at P4, P3 and P1 ("P4P3P1"). In SEQ-P3, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 ("P4"), at P4 and P2 ("P4P2"), and at P4, P2 and P1 ("P4P2P1"). Each subject is assigned to a category with ER and 1-deviation cutoff. The solid bars give the actual type distribution. The checked and stripe bars give the simulated type distribution assuming that these subjects are Lkb or Lka. In certain cases the behaviors of Lkb and Lka are not distinguishable, and they are represented by the same checked bar.

Figure 4: Behavior pattern of each Lk type in sequential games

In theory, these subjects' behavior should follow Table 3 if their belief levels and ability levels remain the same in both simultaneous and sequential games. However, one could be misidentified to a different type if he has a high deviation rate. In order to determine how many misidentifications from the predicted categories could occur, for each of the $L1$-$L4$ types, I simulate the choices of 10,000 pseudo-subjects in sequential games, assuming that all of them are either Lkb or Lka. The average deviation rate of each type used in the simulation is from the type classification results of the simultaneous games in Section 6.2. In Figure 4, the patterned bars of each category give the predicted behavior pattern of Lkb or Lka obtained from simulation. According to the simulation, around 85% of the subjects should fall into the predicted categories if their belief and ability do not change.

Let's first take a look at the $L1$ and $L4$ types, and $L2$ in SEQ-P2. The predictions of Lkb or Lka behavior are the same for these subjects. Their actual choices in SEQ-P2 and SEQ-P3 share similar patterns with the simulated distributions, though less concentrated on the theoretically predicted categories, possibly due to the presence of inconsistent subjects. According to Table 7, exact tests of goodness-of-fit show that in three of these five cases ($L1$ in SEQ-P3, $L2$ in SEQ-P2 and $L4$ in SEQ-P3) the actual type distributions are not different statistically from the predicted ones at the significance level of 0.05 (Table 7). In the case of $L4$ subjects in SEQ-P2, the difference is at the 0.05 level but not the 0.01 level.

Since there might exist both $L2b$ and $L2a$ in SEQ-P3, and both $L3b$ and $L3a$ in SEQ-P2 and SEQ-P3, the type distributions of the $L2$ and $L3$ subjects could be different from the simulated ones. However, the difference should only be reflected in the predicted categories. Specifically, for $L2$'s behavior in SEQ-P3, the difference from the simulated distribution should only be in best response rates at Player 2 position. Therefore small p-values should only occur in the "P4" and "P4P2" categories and the proportions of the other categories should not differ too much from the simulated ones. Similarly, for $L3$'s behavior in SEQ-P2 and SEQ-P3, the difference should only be in best response rates at Player 1 position (reflected in Categories (2) and (3)). The follow-up test of each of the four categories confirms that very few subjects fall outside of the predicted categories.

Thus it could be concluded that the simulated type distributions are good predictions of the aggregate pattern, which provides strong evidence that most subjects are consistent in belief and ability levels.

Result 3 No difference has been found in $L3$'s behavior in SEQ-P2 and SEQ-P3, suggesting that most $L3b$ subjects believe that their opponents are $L2b$ rather than $L2a$.

For the $L3b$ subjects, a remaining question is whether they believe that the opponents are bounded by 2nd-order belief or bounded by 2 steps of reasoning. In SEQ-P2, in order to play the iterated dominant strategy as Player 1, $L3b$ needs to believe that the opponents think two steps, while in SEQ-P3 $L3b$ needs to believe that the opponents use three steps of thinking. If all $L3b$

	SEQ-P2	all categories	each category vs. the rest			
			(1)	(2)	(3)	(4)
			P4	P4P3	P4P3P1	$L0$ + UI
$L1$	vs. predicted-$L1b$ & $L1a$	**0.0012**	**0.0040**	**0.0121**	0.0485	0.6249
$L2$	vs. predicted-$L2b$ & $L2a$	0.1702	0.7124	0.0704	0.5396	0.0648
$L3$	vs. predicted-$L3b$	**0.0000**	0.7669	**0.0000**	**0.0000**	0.2674
	vs. predicted-$L3a$	**0.0000**	0.5337	**0.0000**	**0.0000**	0.2674
$L4$	vs. predicted-$L4b$ & $L4a$	**0.0320**	0.7208	0.0329	**0.0093**	0.4522
	SEQ-P3	all categories	each category vs. the rest			
			P4	P4P2	P4P2P1	$L0$ + UI
$L1$	vs. predicted-$L1b$ & $L1a$	0.0824	0.1752	0.1111	0.4373	0.6249
$L2$	vs. predicted-$L2b$	**0.0000**	**0.0000**	**0.0000**	0.2676	0.3278
	vs. predicted-$L2a$	**0.0000**	**0.0000**	**0.0000**	0.0547	0.6334
$L3$	vs. predicted-$L3b$	**0.0000**	0.0791	**0.0006**	**0.0000**	0.2674
	vs. predicted-$L3a$	**0.0000**	0.5058	**0.0000**	**0.0000**	0.2674
$L4$	vs. predicted-$L4b$ & $L4a$	0.1842	0.1181	1.0000	0.0913	0.4453

Note: the numbers in bold fonts represents the 0.05 significance level test result, with the null hypothesis that the actual data and simulated data are drawn from the same distribution. The first column shows the original tests including all four categories. Columns (2)-(5) show the follow-up tests of each category vs. the sum of all the other categories. For the four follow-up tests, the significance level is corrected by $0.05/4 = 0.0125$.

Table 7: p-values of the exact tests of goodness-of-fit: actual vs. predicted distributions

subjects hold the belief that their opponents are $L2b$, then SEQ-P3 and SEQ-P2 should make no difference for them, which serves as the null hypothesis in this test.

Under the null hypothesis, there should be the same proportion of $L3$ subjects identified as $L3b$ in SEQ-P2 and SEQ-P3. Otherwise, if a subject believes that the opponents are $L2a$, he will not comply with iterated dominance as Player 1 in SEQ-P3. In this case, less $L3$ subjects would be identified as $L3b$ in SEQ-P3 than in SEQ-P2.

In the actual data, no difference could be found statistically in the type distributions from the two sets of sequential games (see **??**). A closer look at the behavior patterns of the $L3$ subjects (Table 8) has confirmed that most of these subjects behave consistently as $L3b$ or $L3a$ in both SEQ-P3 and SEQ-P2. No evidence is found to reject the hypothesis that $L3b$ believes that their opponents are always best responding to $L2$ belief.

It should be noted that with a sample size of 39 $L3$ subjects, to get a power higher than 0.8, it requires at least 25% of them responding to the belief that the opponents are bounded by two

		behavior in SEQ-P2			
		P4	P4P3	P4P3P1	UI
in SEQ-P3	P4	3	2	1	0
	P4P2	0	9	4	1
	P4P2P1	0	5	10	1
	UI	0	1	1	1

Table 8: Behavioral patterns of the 39 $L3$ subjects in the two sequential games

steps of thinking[12]. So there may very well exist a small number of such subjects, who could not be detected in this experiment due to lack of power.

6.4 CRT Scores and k-Levels

This subsection explores the correlation between subjects' identified reasoning ability from the dominance-solvable games and a measure of their cognitive ability. The Cognitive Reflection Test is used as a quick measure of cognitive ability in this experiment. If the $L2b$ ($L3b$) subjects are more capable of reasoning than the $L2a$ ($L3a$) subjects as predicted in the theoretical model, the difference might also be reflected in their CRT scores.

The Cognitive Reflection Test is designed by Frederick (2005) to test people's cognitive ability in decision making. It is composed of three short questions as follows:

(a) A bat and a ball cost 1.10 in total. The bat costs a dollar more than the ball. How much does the ball cost? ___ cents.

(b) If it takes 5 machines 5 min to make 5 widgets, how long would it take 100 machines to make 100 widgets? ___ min.

(c) In a lake, there is a patch of lily pads. Every day, the patch doubles in size. If it takes 48 days for the patch to cover the entire lake, how long would it take for the patch to cover half of the lake? ___ days.

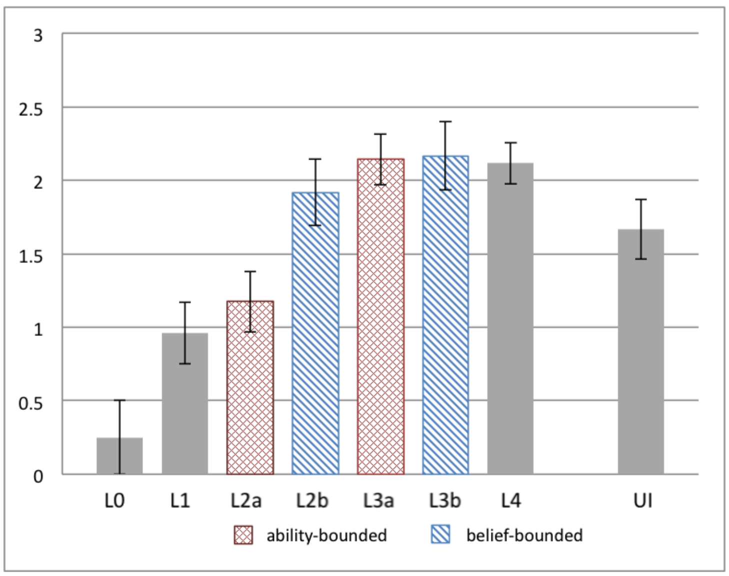
All three questions have intuitive but erroneous answers, and one needs to reflect on them for a brief moment to get them right.

Researches have found correlations of over 0.4 between CRT scores and other cognitive ability measures, such as the SAT, the Wonderlic Personnel Test, and the Vocabulary and Matrix Reasoning subtests (Frederick, 2005; Obrecht et al., 2009; Toplak et al., 2011). The CRT is also reported to relate to some important decision making characteristics. The subjects who score

[12]This result is calculated by G*Power. See Faul et al. (2007).

high in the CRT are more patient, and are more willing to take risks (Frederick, 2005). In addition, because the CRT tasks separate impulsive and reflective thinkers, it suggests that the ones who score higher in the test tend to do more rational thinking and are less likely to succumb to heuristics and bias (Oechssler et al., 2009).

The participants in this experiment obtain a mean CRT score of 1.64, which is comparable to the results in the literature. In Figure 5 the mean CRT scores are displayed by types. Among the lower types ($L0$-$L2b$), subjects' CRT scores increase with their identified levels. Two-sided t-tests show that the differences are significant between these groups, except for $L1$ and $L2a$. The most striking result is that the $L2a$ subjects, who are identified as being of lower reasoning abilities, did much worse in CRT than the $L2b$ subjects, although both types exhibited the same behavior in the simultaneous games. The results of CRT and type identification corroborate each other, suggesting that cognitive ability plays an important role in strategic reasoning.



Note: Standard errors are reported in the error bars.

Figure 5: Average CRT scores by types

The differences between high types ($L2b$-$L4$) are not that clear. T-tests show that the means are not different for these four groups. All of these subjects are able to do at least three steps of iterative thinking in the ring games, and their mean scores are pretty high[13]. Therefore, one possible explanation of why higher types are not separable by CRT scores could be that the CRT results do not distinguish between the subjects whose reasoning abilities have reached a certain high level.

[13]To serve as a comparison, Fredrick (2005) conducted the CRT on 3,428 subjects, primarily college students, and found a mean score of 1.24. The mean scores from top universities like Princeton and Harvard are around 1.4-1.6. Only MIT students get a high mean of over 2.

Another interesting observation is the difference between $L0$ and unidentified subjects. The mean CRT score of $L0$ subjects is close to 0. Actually, 7 out of 8 subjects in this group scored 0 in the test. Contrastingly, the unidentified group performs much better and their average score is not statistically different from the mean of the whole sample. It supports the separation of these unidentified subjects from the irrational $L0$. The unidentified subjects could be of high cognitive ability, but do not follow the prediction of the level-k model. Or it could be that they are inconsistent and act as different types across different games.

6.5 Robustness Check: Learning Effect

An essential assumption in the above analysis is that each subject's belief bound and ability bound remain constant throughout the experiment, with a few deviations due to random preference shift or trembling hand. However, if certain learning effects prevailed, the subjects became more proficient in identifying iterated dominant strategies as the experiment proceeded. Then the observed pattern could be driven by the order of games in the experiment. For example, the subjects who played the more difficult positions, such as Player 1 or Player 2 positions, in the later periods would have a larger chance of best responding. In this case, the higher types might not be the group with higher ability, but the group with more opportunity to learn.

A detailed analysis of learning effects will be reported in Appendix B. In summary, I find mild learning effects at a couple of player positions, but the type classification results do not appear to be affected by learning. This subsection compares the type distributions of the whole sample and the groups who might have an advantage in learning, showing that learning has almost no effect in shaping the type distribution.

Given the special structure of the ring games, it might be advantageous to play the Player 4 positions first, for it could help the subjects to figure out early in the experiment that the games could be solved by iterated dominance. In addition, playing as second-movers might also help, for it motivates the subject to look into the dependency relationships between the opponents and himself. If this is true, then the subjects who played more Player 4 positions or second-movers in the earlier stage are more likely to behave as higher types. To address this concern, I check whether the performances of these subjects differ from the whole sample.

In each session, subjects played the 24 games in different random orders. Table 9 gives the number of subjects who played more Player 4 positions and second-mover positions in the earlier 12 games. The numbers of Player 4 positions are denoted by $n(P4)$, and the number of second-mover positions by $n(PSM)$. There are 44 subjects who played at more than four Player 4 positions in the earlier 12 games, and 53 subjects who played at more than six Player 4 or second-mover positions. These subjects are regarded as having advantages in learning. The following results still hold if other cutoffs are used to determine advantages.

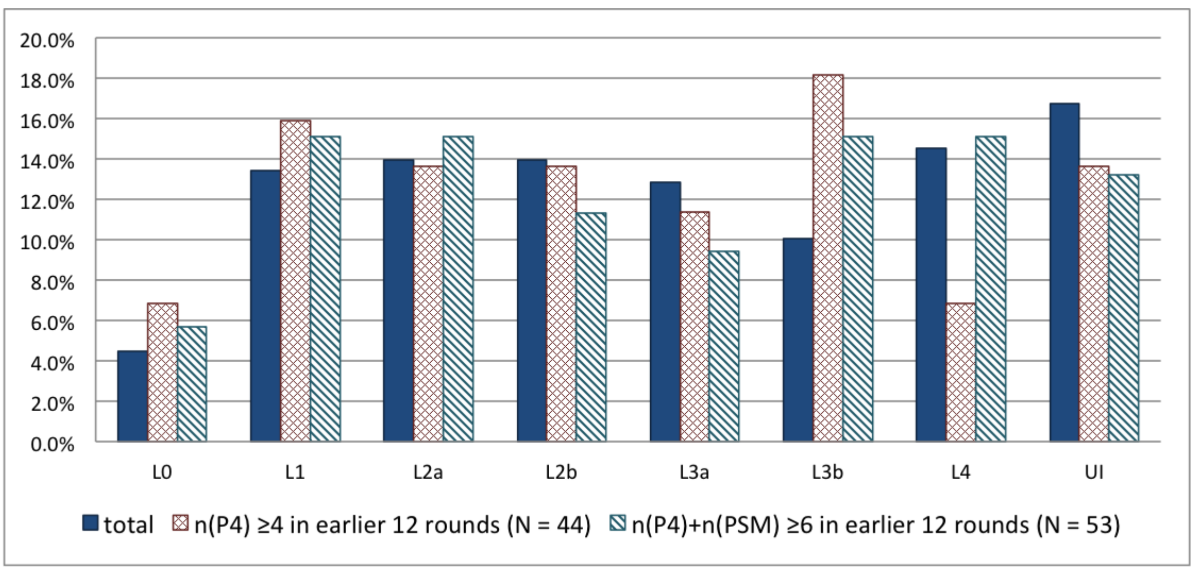
As shown in Figure 6, no evidence has been found that they performed better than the rest

$n(P4)$	≥ 1	≥ 2	≥ 3	$\geq$ **4**	≥ 5	≥ 6
N	179	155	124	**44**	31	0
$n(P4) + n(PSM)$	≥ 3	≥ 4	≥ 5	$\geq$ **6**	≥ 7	≥ 8
N	179	154	108	**53**	36	7

Note: $n(P4)$ denotes the number of Player 4 positions played by the subject in the earlier 12 games. $n(PSM)$ denotes the number of second-mover positions played by the subject in the earlier 12 games. The bold fonts represent the cutoffs used in the following analysis.

Table 9: Number of subjects who played more advantageous positions in earlier 12 games

of the subjects. The distributions of these groups are not shifted toward higher types. The goodness-of-fit tests show that the type distribution of the subjects with $n(P4)+n(PSM) \geq 6$ is not statistically different from the distribution of the whole sample, and that, for the group with $n(P4) \geq 4$, the differences only occur in the $L3b$ and $L4$ categories. Therefore it is safe to say that the identified patterns of the main results are not affected by subjects' learning of iterated dominance.



Note: $n(P4)$ denotes the number of player 4 positions played by the subject in the earlier 12 games. $n(PSM)$ denotes the number of second-mover positions played by the subject in the earlier 12 games.

Figure 6: Type distributions of the subjects who have an advantage in learning

7 Conclusion

Existing literature on the level-k model focuses on identifying people's reasoning steps in strategic situations. This study seeks to identify the determinants of the observed reasoning steps. The experiment separates the belief-bounded subjects (Lkb) who behave as Lk due to their beliefs and the ability-bounded subjects (Lka) who stop at level-k because they could think, at most, k steps. The separation happens at certain first-mover positions of the sequential ring games, where it requires three or four steps to respond to $L2$ or $L3$ belief. The $L2b$ or $L3b$ subjects are still able to best respond to the same $L2$ or $L3$ belief as they did in the simultaneous games. But the $L2a$ or $L3a$ ones could do two or three steps of reasoning at most, and thus could not handle the one more step.

Most subjects behave consistently, in terms of belief bounds and ability bounds, across the three sets of simultaneous and sequential ring games. Out of the 50 and 39 subjects classified as $L2$ and $L3$ from their choices in the simultaneous games, around half have reached their upper boundaries of reasoning. In addition, evidence on $L3$ subjects supports that they believe that the opponents are belief-bounded instead of ability-bounded. Finally, the CRT scores are significantly higher for the high ability $L2b$ than the low ability $L2a$, which supports the separation of the two types. But higher types ($L2$, $L3a$, $L3$ and $L4$) are not distinguishable using CRT scores.

The findings suggest large heterogeneity in subjects' abilities to best respond to even low order belief. Thus, the observed low levels in the previous studies could be explained by both the presence of low ability types and the low-order beliefs of high ability types. Although the high types have incorrect beliefs, their low-order beliefs are not entirely unfounded, given the large proportion of the cognitively bounded subjects.

The existing literature has demonstrated the descriptive power of the level-k model. To make it also an explanatory and predictive model, it requires a better understanding of why people behave at certain levels, or where people get their belief from. For example, the existence of ability-bounded subjects in this study shows that a lot of people might not start with a clear idea of the opponents' levels. Rather, their belief could be formed through the anchoring and adjusting process suggested by Brandenburger and Li (2015), and this process would stop when they have reached their cognitive boundaries. Therefore, one direction of future works could be to explore the relationship between players' ability bounds and belief formation.

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APPENDIX

A Type Classification of Lkb and Lka with Alternative Cutoffs

		Baseline + Sequential Types								
		$L0$	$L1$	$L2a$	$L2b$	$L3a$	$L3b$	$\geq L4$	UI	sum
Baseline Types	L0	**6**	0	0	0	0	0	0	1	7
	L1	0	**13**	1	0	0	1	0	11	26
	L2	0	0	**15**	**18**	0	0	0	17	50
	L3	1	0	0	0	**15**	**13**	0	10	39
	L4	0	0	0	0	0	0	**25**	5	30
	UI	1	0	0	0	0	0	0	26	27
	sum	8	13	16	18	15	14	25	70	179

Note: $N = 179$. Subjects are assigned to a type with, at most, 2 deviations. Otherwise they are classified as $L0$ or unidentified. The subjects classified as unidentified are able to choose 5 out of 6 dominant strategies as player 4 but do not match any of the predicted patterns. The numbers in bold fonts represent the subjects who remain in the same Lk category in the two type classifications.

Table 10: Type assignment according to the observations from SIMUL, SEQ-P3 and SEQ-P2 (2 deviations)

		Baseline + Sequential Types								
		$L0$	$L1$	$L2a$	$L2b$	$L3a$	$L3b$	$\geq L4$	UI	sum
Baseline Types	L0	**6**	1	0	0	0	0	0	0	7
	L1	0	**23**	1	1	0	1	0	0	26
	L2	0	4	**24**	**21**	0	1	0	0	50
	L3	1	0	1	0	**21**	**15**	0	1	39
	L4	0	0	1	1	1	0	**26**	1	30
	UI	1	6	4	4	0	2	0	10	27
	sum	8	34	31	27	22	19	26	12	179

Note: $N = 179$. Subjects are assigned to a type with, at most, 4 deviations. Otherwise they are classified as $L0$ or unidentified. The subjects classified as unidentified are able to choose 5 out of 6 dominant strategies as player 4 but do not match any of the predicted patterns. The numbers in bold fonts represent the subjects who remain in the same Lk category in the two type classifications.

Table 11: Type assignment according to the observations from SIMUL, SEQ-P3 and SEQ-P2 (4 deviations)

B Additional Analysis on Learning Effects

A thorough analysis on learning effects is reported in this section. I start by checking whether there exist systematic type shifts in the data. That is, whether subjects are more likely to shift to a higher (lower) type in the later (earlier) period of the experiment. Since the choices in all 20 first-mover positions are needed to pin down a subject's type, it is not possible to estimate one's type in the early and late parts of the experiment separately. Instead, I look at the deviations from the choice pattern of one's assigned type. The deviation observed in the type assignment process could be sorted into one of three cases: (1) a non-equilibrium strategy is chosen at a position where the type should have chosen an equilibrium strategy. (2) two different strategies are chosen at the same position of the two paired rings where the type should be on off-equilibrium path and choose the same strategy, and at least one of the two strategies chosen is an equilibrium strategy. (3) two different strategies are chosen at the same position of the two paired rings where the type should be on off-equilibrium path and choose the same strategy, and neither of the two strategies chosen is an equilibrium strategy.

Case (1) implies that the subject might shift to a lower level when playing that game and Case (2) corresponds to the shift to a higher level. If learning affects subjects' behavior, they should be more likely to deviate to a higher level in the later half of the experiment and more likely to a lower level in the earlier half. If, however, the growth of fatigue plays a more important role, it should be opposite. Of course it could not be ruled out that some deviations in Case (1) and (2) are caused by preference shifts or mistakes. But if the occurrences of preference shifts or mistakes are assumed to be time-invariant, then they could be canceled out when only the differences of the earlier and later halves are examined.

	8 Baseline Games			20 first-mover Games		
	Case (1)	Case (2)	Case (3)	Case (1)	Case (2)	Case (3)
Shifts	downward	upward	-	downward	upward	-
Earlier	12	26	25	42	80	53
Later	2	27		24	79	

Note: the first two cases are counted in the earlier and later 12 games respectively. $L0$ and unidentifiable subjects are excluded.

Table 12: Deviations sorted into three cases

Table 12 reports the deviations of the classified subjects, from both the 8 SIMUL games and the 20 first-mover games type assignments. $L0$ and unidentifiable subjects are excluded from this analysis, because according to the definition $L0$s could be using any combination of strategies, and since I could not identify the decision rules of the unidentifiable it would be hard to determine which choices are deviations from their rules. The 24 games were played in a random order and the orders of play were different for each subject. Cases (1) and (2) could therefore be put into two categories, that is whether these deviations occur in the first or later 12 games. In Case (3) it could only be observed that subjects are choosing two different strategies at the same position, but it is impossible to tell which one is the deviation (or both could be deviations). So only the total numbers of deviations in Case (3) are reported.

In Case (1), players deviate to a lower type. This kind of deviation is more likely to happen in the first half of the experiment, suggesting some sort of learning effects. However, the occurrences of Case (2) deviations, which imply a shift to a higher level, are quite close between earlier and later periods of the experiment.

I next run a probit regression to determine the learning effect specifically at each position.

$$\text{Probit}(Y_i) = \alpha + \beta_1 \text{L12}_i + \beta_2 \text{POS}_i + \beta_{3j} \text{POS}_{ij} \times \text{L12}_i + \epsilon_i, \tag{2}$$

where $Y_i = 1$ when an equilibrium strategy is chosen, and $Y_i = 0$ otherwise; $L12_i = 1$ if that choice is made in the later 12 games, and $L12_i = 0$ otherwise; POS_{ij} denotes the position dummy at position j. The session fixed effects are also controlled.

		player 1	player 2	player 3	player 4
Baseline	G1	0.215	0.009	0.287	0.127
		(0.193)	(0.189)	(0.210)	(0.389)
	G2	0.267	0.055	0.235	0.010
		(0.190)	(0.188)	(0.211)	(0.417)
		player 1	player 2 (2nd)	player 3	player 4
Seq-P2	G3	0.210	0.108	0.570**	-0.226
		(0.193)	(0.331)	(0.234)	(0.365)
	G4	0.232	-0.683	-0.103	0.272
		(0.199)	(0.427)	(0.227)	(0.315)
		player 1	player 2	player 3 (2nd)	player 4
Seq-P3	G5	0.028	0.492**	0.142	-
		(0.187)	(0.200)	(0.344)	-
	G6	0.393**	-0.144	0.237	0.010
		(0.193)	(0.197)	(0.369)	(0.395)

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Standard errors are reported in the parentheses. (2nd) denotes second stage movers. Player 4 of G5 is dropped due to collinearity.

Table 13: Experience effect at each position ($\beta_1 + \beta_{3j}$)

Table 13 reports the coefficients $\beta_1 + \beta_{3j}$ of each position j. One position dummy, player 4 of G5, is dropped because of collinearity. Significant positive learning effects are found at only 3 of the 23 positions.

Since the identification uses at least a pair of ring games, the learning effects at three positions is unlikely to affect the type distribution. It is reported in Section 6.5 that the performance of the subjects who could have better opportunities to learn is not different from the whole sample. Here I further show with multilogit regressions that playing more player 4 positions or second-mover positions in the earlier periods does not affect the probability of being assigned to a high type (Table 14 and Table 15).

Independent Variable: $I(n(P4) > 4$ in earlier 12 games)							
base outcome	$L0$	$L1$	$L2a$	$L2b$	$L3a$	$L3b$	$L4$
vs $L0$		-0.398	-0.713	-0.770	-0.843	0.182	-1.713*
		(0.883)	(0.897)	(0.887)	(0.893)	(0.884)	(0.999)
vs $L1$	0.398		-0.315	-0.372	-0.445	0.581	-1.315
	(0.883)		(0.720)	(0.670)	(0.715)	(0.656)	(0.823)
vs $L2a$	0.713	0.315		-0.0564	-0.129	0.896	-1.000
	(0.897)	(0.720)		(0.682)	(0.703)	(0.713)	(0.832)
vs $L2b$	0.770	0.372	0.0564		-0.0730	0.952	-0.943
	(0.887)	(0.670)	(0.682)		(0.697)	(0.676)	(0.811)
vs $L3a$	0.843	0.445	0.129	0.0730		1.025	-0.870
	(0.893)	(0.715)	(0.703)	(0.697)		(0.723)	(0.835)
vs $L3b$	-0.182	-0.581	-0.896	-0.952	-1.025		-1.895**
	(0.884)	(0.656)	(0.713)	(0.676)	(0.723)		(0.801)
vs $L4$	1.713*	1.315	1.000	0.943	0.870	1.895**	
	(0.999)	(0.823)	(0.832)	(0.811)	(0.835)	(0.801)	

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Standard errors are reported in the parentheses. Each row shows a regression with a different type being the base outcome. Session fixed effects are included in the multilogit regressions. Unidentifiable subjects are excluded.

Table 14: Multilogit regression of types on Learning Effects (1)

Independent Variable: $I(n(P4) + N(PSM) > 6$ in earlier 12 games)							
base outcome	$L0$	$L1$	$L2a$	$L2b$	$L3a$	$Lb3$	$L4$
vs $L0$		0.381	-0.320	-0.633	-0.740	0.689	-0.761
		(0.893)	(0.907)	(0.926)	(0.948)	(0.951)	(0.922)
vs $L1$	-0.381		-0.700	-1.014	-1.121	0.308	-1.142*
	(0.893)		(0.690)	(0.681)	(0.746)	(0.704)	(0.674)
vs $L2a$	0.320	0.700		-0.314	-0.420	1.009	-0.441
	(0.907)	(0.690)		(0.699)	(0.740)	(0.758)	(0.697)
vs $L2b$	0.633	1.014	0.314		-0.107	1.322*	-0.128
	(0.926)	(0.681)	(0.699)		(0.758)	(0.757)	(0.696)
vs $L3a$	0.740	1.121	0.420	0.107		1.429*	-0.0208
	(0.948)	(0.746)	(0.740)	(0.758)		(0.819)	(0.741)
vs $L3b$	-0.689	-0.308	-1.009	-1.322*	-1.429*		-1.450*
	(0.951)	(0.704)	(0.758)	(0.757)	(0.819)		(0.742)
vs $L4$	0.761	1.142*	0.441	0.128	0.0208	1.450*	
	(0.922)	(0.674)	(0.697)	(0.696)	(0.741)	(0.742)	

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Standard errors are reported in the parentheses. Each row shows a regression with a different type being the base outcome. Session fixed effects are included in the multilogit regressions. Unidentifiable subjects are excluded.

Table 15: Multilogit regression of types on Learning Effects (2)

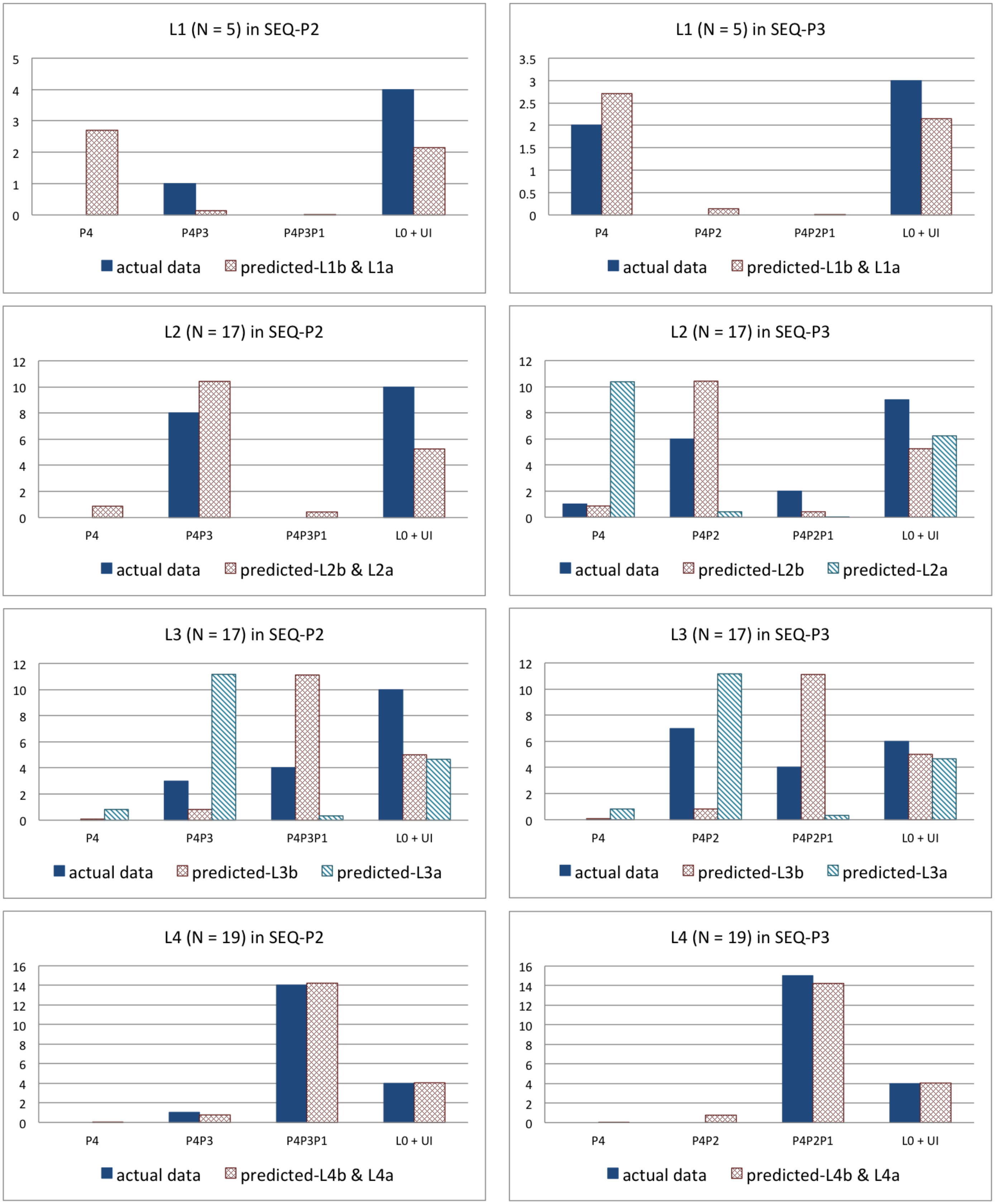
C Other Results

C.1 Type Switch Pattern with Alternative Cutoffs

Table 5 has shown what the type distribution looks like allowing for 0 or 2 deviations. If no deviation is allowed for during the type assignment process, there will be a large fraction of subjects (113 out of 184) who could not be put into any of the $L0$-$L4$ categories. With the 0-deviation cutoff, since few subjects' behavior profile could be matched to a certain type with 0 deviation in all three sets of games, it is hard to determine a clear pattern of each type's behavior in the sequential games. As shown in Figure 7, although most $L4$ subjects behave as predicted in both sequential games, half of the $L1$, $L2$ and $L3$ subjects fall into the unidentified category. So it is less clear what proportion of the subjects are best responding to their belief and what proportion are bounded by reasoning ability in the sequential games.

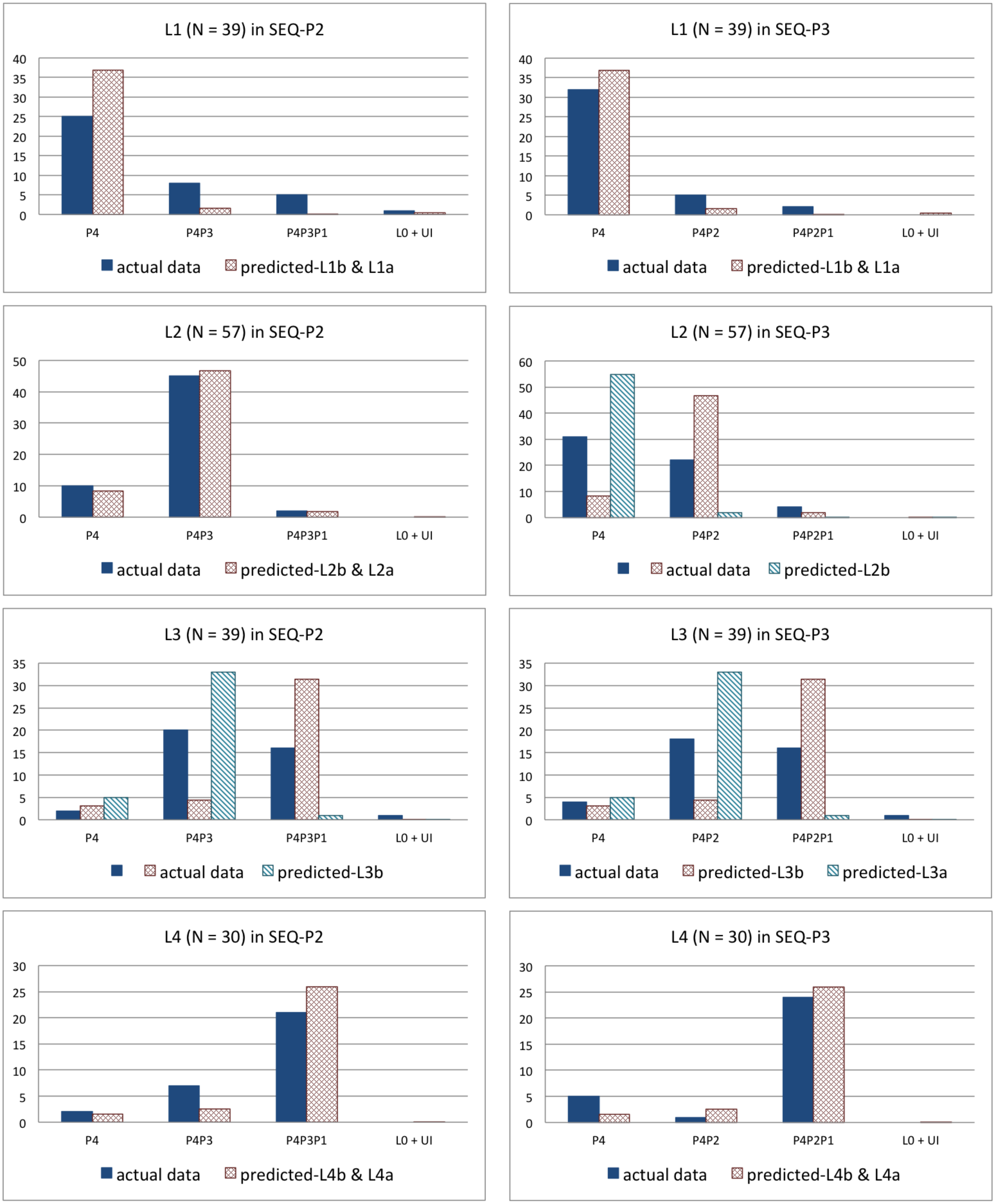
When allowing for 2 instead of 1 deviation in the matching process, the risk of misidentifying the randomly choosing subjects to an Lk type increases. But it is still worth looking at the results with the 2-deviation cutoff, in which almost all the subjects could be classified.

Most previously unidentified subjects in the simultaneous games are moved to one of the lower types ($L1$ and $L2$) when allowing for 2 deviations, and there is no change in the number of $L3$ and $L4$ subjects. The behavioral pattern of each type in the sequential games appears to be similar as in the main analysis (Figure 8). But it should be noted that since it identifies more lower types from the previously unidentifiable pool by using the 2-deviation cutoff, the use of this cutoff picks up slightly high proportion of the ability-bounded $L2a$ and $L3a$ subjects.



Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterated) dominant strategies at P4 ("P4"), at P4 and P3 ("P4P3"), and at P4, P3 and P1 ("P4P3P1"). In SEQ-P3, subjects are sorted into three categories: choosing the (iterated) dominant strategies at P4 ("P4"), at P4 and P2 ("P4P2"), and at P4, P2 and P1 ("P4P2P1"). Each subject is assigned to a category with ER and 0-deviation cutoff. The solid bars give the actual type distribution. The checked and stripe bars give the simulated type distribution assuming that these subjects are Lkb or Lka. In certain cases the behaviors of Lkb and Lka are not distinguishable, and they are represented by the same checked bar.

Figure 7: Behavioral patterns in sequential games (0 deviation)



Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 ("P4"), at P4 and P3 ("P4P3"), and at P4, P3 and P1 ("P4P3P1"). In SEQ-P3, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 ("P4"), at P4 and P2 ("P4P2"), and at P4, P2 and P1 ("P4P2P1"). Each subject is assigned to a category with ER and 2-deviation cutoff. The solid bars give the actual type distribution. The checked and stripe bars give the simulated type distribution assuming that these subjects are *Lkb* or *Lka*. In certain cases the behaviors of *Lkb* and *Lka* are not distinguishable, and they are represented by the same checked bar.

Figure 8: Behavioral patterns in sequential games (2 deviations)

C.2 Type Assignment with the Assumption of Uniform Randomizing $L0$

The assumption used to predict Lk behavior in the ring games is that an Lk player does not respond to the changes in $(k+1)$- or higher direct opponents' payoffs (**A1**). An alternative assumption, which is widely used in Lk experiments, is that the irrational $L0$ type uniformly randomize on all the possible actions (**UP**: uniform prior on $L0$).

		$L0$	$L1$	$L2$	$L3$	$L4$	UI
SIMUL	ER	7	26	50	39	30	27
		3.9%	14.5%	27.9%	21.8%	16.8%	15.1%
	UP	6	10	43	39	53	28
		3.4%	5.6%	24.0%	21.8%	29.6%	15.6%
		$L0$	P4	P4P3	P4P3P1	-	UI
SEQ-P2	ER	7	43	69	46	-	14
		3.9%	24.0%	38.5%	25.7%	-	7.8%
	UP	7	32	60	78	-	2
		3.9%	17.9%	33.5%	43.6%	-	1.1%
		$L0$	P4	P4P2	P4P2P1	-	UI
SEQ-P3	ER	5	68	46	45	-	15
		2.8%	38.0%	25.7%	25.1%	-	8.4%
	UP	5	22	46	94	-	12
		2.8%	12.3%	25.7%	52.5%	-	6.7%

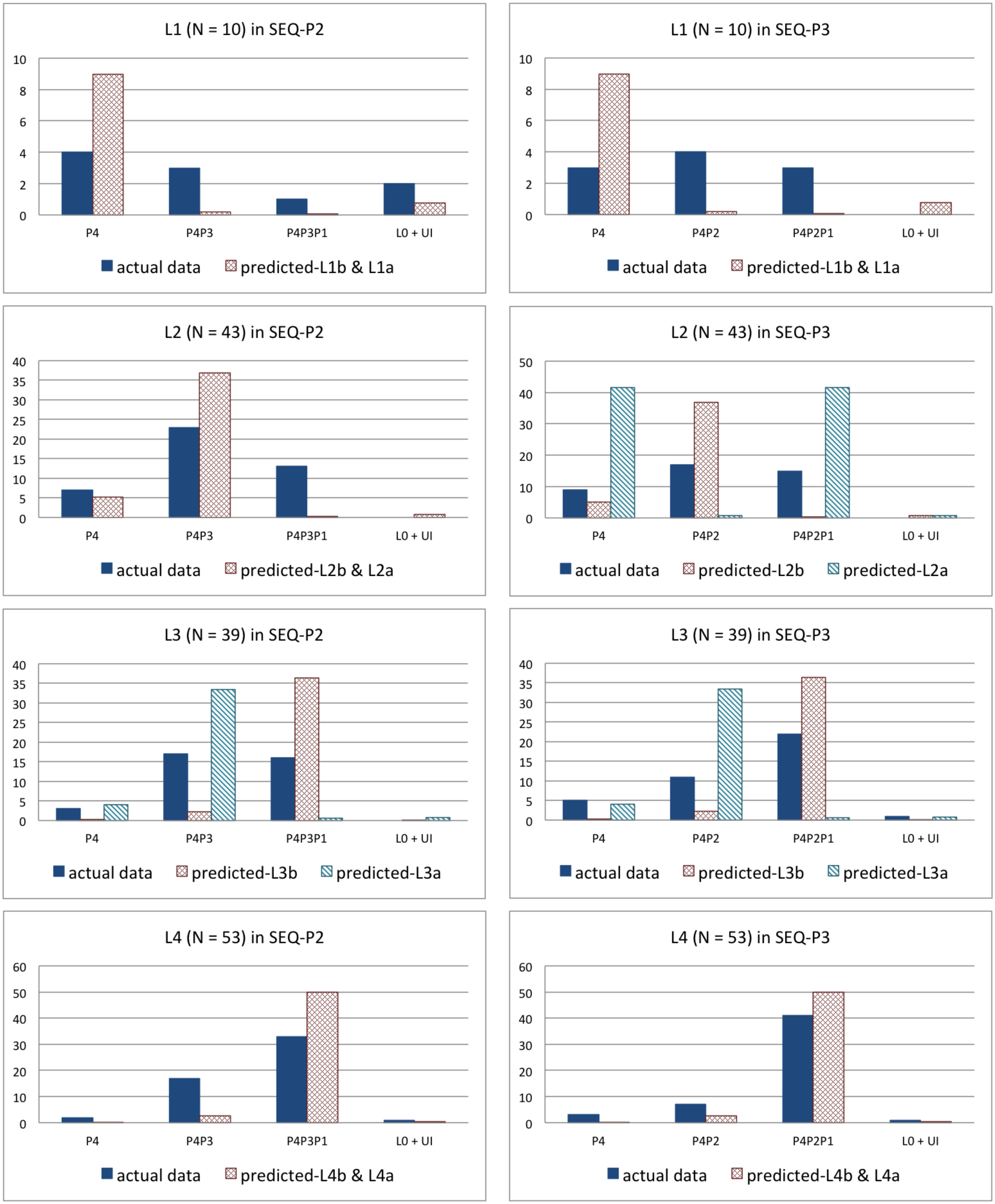
Note: $N = 179$. Subjects are assigned to a type with no more than 1 deviation when using $A1$, with no more than 2 deviations when using UP. Otherwise they are assigned to $L0$ or the unidentified group. The subjects classified as unidentified play dominant strategies as Player 4 but do not match any of the predicted patterns.

Table 16: Type assignment using the two assumptions ER and UP

The distribution of assigned types using the predicted Lk behavior of UP is given in Table 16. The 2-deviation cutoff is used here, so a random choosing subject only has a probability of less than 5% of being assigned to one of the Lk types, which is comparable to the main results. The distributions shift to the right in all three sets of games. In the simultaneous games, the number of $L4$ subjects almost doubles, and the number of $L1$ drops by more than a half. The shift is even larger in SEQ-P3. The number of subjects assigned to "P4" category under UP is less than a third of the number under $A1$. And the number in "P4P2P1" category increases by more than 100%. A closer look at the type changes confirms that everyone's level rises or at least stays the same. Nobody goes to a lower level under UP. Nevertheless, the numbers of $L0$ and the unidentified are quite close. There are 6 subjects classified as $L0$ and 14 classified as unidentified under both assumptions.

I try to perform the same analysis of each type's behavioral pattern in the sequential games (Figure 9). However, the overestimation of lower types in the two sequential games makes the treatment effects less clear. A large number of $L1$- and $L2$-behaving subjects in SIMUL are classified as "P4P2P1" in the sequential games, which is inconsistent with the theoretical prediction. It is impossible to tell how much subjects are Lka with such large inconsistency across the three sets of games. The overestimation is less severe among higher types, and a similar pattern could be observed on $L3$ and $L4$ subjects as in the main analysis.

Type classification with UP tends to overestimate the lower types. This is because the predicted action profile of each type on the off-equilibrium path using UP is a special case of that using $A1$. Since UP puts stronger restrictions on the off-equilibrium path, any subject has a smaller chance to be assigned to a low type. The overestimation would be more severe if fewer subjects follow the prediction of UP, as observed in SEQ-P3. Hence I use $A1$ in the main analysis, which I believe provides more robust and reliable results.



Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 ("P4"), at P4 and P3 ("P4P3"), and at P4, P3 and P1 ("P4P3P1"). In SEQ-P3, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 ("P4"), at P4 and P2 ("P4P2"), and at P4, P2 and P1 ("P4P2P1"). Each subject is assigned to a category with UP and 2-deviation cutoff. The solid bars give the actual type distribution. The checked and stripe bars give the simulated type distribution assuming that these subjects are *Lkb* or *Lka*. In certain cases the behaviors of *Lkb* and *Lka* are not distinguishable, and they are represented by the same checked bar.

Figure 9: Behavioral patterns with the uniform prior assumption (UP)