Does Level-k Behavior Imply Level-k Thinking?

Ye Jin*

April 25, 2016

Abstract

I design an experiment to interpret the observed Lk behavior. It distinguishes between the "Lkb" players, who have high ability and best respond to Lk belief, and the "Lka" players, who could use, at most, k steps of reasoning, and thus could not respond to $L(k+1)$ or higher-order belief. The separation utilizes a combination of simultaneous and sequential ring games. In the sequential games it requires more than k reasoning steps to respond to Lk belief, so Lkb players still best respond but Lka would fail. I find that around half of the $L2$ and $L3$ subjects are best responding to $L2$ or $L3$ belief, while the rest have reached their upper boundaries of reasoning. Additionally, subjects' CRT scores, a measure of their cognitive ability, support the separation of the two types. The findings suggest that both belief and reasoning ability could be the decisive factors of players' observed levels.

1 Introduction

The level-k theory is proposed to model player's systematic deviation from Nash equilibrium by allowing for inconsistent beliefs (Nagel, 1995; Stahl and Wilson, 1994, 1995; Ho et al., 1998). The model starts with the irrational or non-strategic $L0$ type, who is usually assumed to choose randomly or use certain salient strategies. $L1$ best responds to the belief that all the rest of the players are $L0$ and each higher Lk best responds to $L(k-1)$.

Current literature on the level-k model mostly focuses on the identification of Lk types by looking at the number of reasoning steps used by the subjects. Most subjects in these experiments appear to be using no more than two or three steps of reasoning when playing a game for the first time. Then a natural question is why do they stop at these low levels?

*Affiliation: Department of Economics, University of California, Berkeley. Email: jiny@berkeley.edu. I thank Shachar Kariv for invaluable advice and constant support throughout this project. I am grateful to Terri Kneeland for her helpful advice and the access of her data and experiment program. I thank David Ahn, Ned Augenblick, Adam Brandenburger, Stefano Dellavigna, Yoram Halevy, Teck H. Ho, John Morgan, Chris Shannon, Aluma Dembo, Michèle Müller-Itten, Yang Xie and all the seminar participants for helpful discussions and comments. I thank the Xlab at UC Berkeley for their assistance during data collection and the research grant. All errors are mine.

This study attempts to identify whether subjects behave at these levels due to the belief that other participants do not think more than one or two steps, or due to their own lack of ability to think further. A subject's exhibited sophistication level in a certain game depends on both his belief in the opponents' rationality levels and his ability to finish all the required reasoning steps. But belief and ability are not directly observable from choice data. In most studies, a subject is classified as Lk if he uses k steps of reasoning, which implies that both his belief level and ability level are greater than or equal to k. But few studies have provided a clear answer to whether belief or ability is the decisive factor of the observed levels.

The original level-k model assumes that the heterogeneous levels are due to different orders of belief. It is argued in some studies that the observed levels are far below the subjects' upper bounds of reasoning ability (for examples, see Crawford and Costa-Gomes (2006) and Arad and Rubinstein (2012)). It implies that people have a high reasoning ability but believe that others are of a much lower level. This interpretation could be justified by the fact that people tend to underestimate their opponents, and that being aware of this fact, the more sophisticated players also have to respond to the belief in low-order belief. Therefore, when given enough incentives or induced higher-order beliefs, the players will proceed to higher levels or even reach equilibrium.

The alternative story says that Lk behavior is the result of bounded reasoning ability. Subjects might have higher-order belief or even equilibrium belief, but their limited ability prevents them from using too many thinking steps. Since existing studies using various classes of games find that the average k-levels are around two, this story suggests that when playing a game for the first time, the average people are only able to do two rounds of iteration, which is far from the requirement of reaching equilibrium in a lot of games. So the two stories have very different implications.

In this study, a Lk player is classified as Lkb if his observed level is determined by his belief, and as Lka if determined by his ability. The identification of Lkb and Lka players utilizes a combination of simultaneous and sequential games. The ring games are first studied in the innovative work of Kneeland (2015). She uses a set of simultaneous ring games to separate different orders of rationality. Here I provide an illustration of the separation of $L2b$ and $L2a$ with 3-player ring games.

Consider a 3-player simultaneous ring game G (Figure 1). Player 1's (P1) payoff depends on his and player 2's (P2) actions. Player 2's payoff depends on his and player 3's (P3) actions. Player 3's payoff depends on his and player 1's actions. In particular, Player 3 has a dominant strategy. Players 1 and 2's iterative dominant strategies could be derived from Player 3's best response, and thus the unique Nash equilibrium (c, c, b) could be solved by three rounds of iterative dominance.

Player 2

Player 1	a	b	c
a	32	24	4
b	36	20	10
c	26	18	16

P1's Payoff

Player 3

Player 2	a	b	c
a	18	20	26
b	14	8	36
c	6	24	28

P2's Payoff

Player 1

Player 3	a	b	c
a	8	12	24
b	20	16	32
c	18	14	20

P3's Payoff

Note: In G, the three players move simultaneously. In G', players 1 and 3 move in the first stage. Player 2 moves in the second stage after observing players 1's and 3's choices.

Figure 1: G and G': 3-player ring games

G is similar to the simultaneous games used in Kneeland (2015). In this game, player 3 only needs to hold at least $L1$ belief and use one step of reasoning to play the dominant strategy. Player 2 needs to believe that player 3 is best responding ($L2$ belief) and use two steps of reasoning to solve for his iterative dominant strategy, and player 1 needs $L3$ belief and three steps of reasoning.

If a player chooses the (iterative) dominant strategies as players 2 and 3, but not as player 1, he would be classified as $L2$. However, it could not be inferred from these simultaneous games whether he is responding to $L2$ belief, or whether he has a higher-level belief but could do, at most, two steps of reasoning. The difficulty of identification lies in the fact that, for a certain choice of action, the required levels of belief and the required number of reasoning steps are always the same. This is also true in most games in the existing literature.

To cope with this problem, I introduce the sequential ring games, in which one needs to use more than k steps to respond to Lk belief. The sequential game G' adopts the same payoff structure as in G, but includes two stages. Players 1 and 3 move simultaneously in the first stage. Player 2 observes their actions and then move in the second stage. The sequence of moves is common knowledge among all players. In this game, solving player 3's problem still requires one step of reasoning. Since player 2 will be able to observe player 3's choice before making his own decision, he also needs to think one step in order to be able to best respond.

It is interesting to study the decision situation of player 1 in the sequential game G'. Obviously, player 1 of G' still needs three steps to arrive at the iterative dominant strategy. But he only needs to believe that player 3 picks the dominant strategy and that, after observing player 3's move, player 2 best responds accordingly, which is $L2$ belief by definition. So as player 1 in G', holding $L2$ belief requires one to think three steps.

Now consider a player who exhibits $L2$ behavior in the simultaneous game G. His behavior as player 1 in G' reveals whether he is Lkb or Lka (Table 1). If he has $L2$ belief and his ability is not binding, then he should believe that both players 2 and 3 will be able to pick the dominant strategies, and choose his iterative dominant strategy as well. However, if he behaves as $L2$ in G because he is bounded by two steps of reasoning, he is not able to use three steps and obey iterative dominance as player 1 of G'. Thus Lkb or Lka could be separated using player 1 position of 3-player ring games. Larger rings are needed to get separation of higher types.

G: simultaneous	player 1	player 2	player 3
$L2b$	$\times$	$\surd$	$\surd$
$L2a$	$\times$	$\surd$	$\surd$
G': sequential	player 1	second mover	player 3
$L2b$	$\surd$	$\surd$	$\surd$
$L2a$	$\times$	$\surd$	$\surd$

Note: $\surd$ denotes choosing the (iterative) dominant strategy at this position, $\times$ otherwise.

Table 1: Separation of $L2b$ and $L2a$

The games used in the actual experiment include 4 players. In each game, player 4 is the one who has a dominant strategy. The games are sorted into three sets, the simultaneous games (SIMUL) and two sets of the sequential games (SEQ-P2 and SEQ-P3). The two simultaneous rings could be used to identify Lk behavior up to $L4$. In the two rings of SEQ-P2, player 2 moves in the second stage, and in the two rings of SEQ-P3, player 3 moves in the second stage. The combination of the simultaneous and sequential games allows me to get separation on the subjects who behave as $L2$ or $L3$ in the simultaneous games. More specifically, a $L2b$ ($L3b$) player best responds to $L2$ ($L3$) belief in both the simultaneous and the sequential games. A $L2a$ ($L3a$) player could use at most two (three) steps of reasoning, and thus is not able to best respond to the same belief in the sequential games.

A total of 184 subjects participated in the experiment, and enough observations were collected from 179 of them to perform the analysis. Each subject played the 8 positions of the two simultaneous games and the 16 positions of the four sequential games, including 4 second mover positions. There are 50 and 39 subjects classified as $L2$ or $L3$ respectively in the simultaneous games. More than half of these subjects failed to best respond to the same belief in the sequential game, suggesting that a considerable amount of subjects are bounded by their ability.

I then perform a subject-by-subject type classification (Figure 2). Out of the 50 $L2$ subjects, 21 are confirmed to be $L2b$, who still best respond to $L2$ belief in the sequential games, and 21 appear to be $L2a$, who could not finish the one more reasoning step in the sequential games (the remaining 8 subjects are classified as $L1$, $L3$ or unidentified). Out of the 39 $L3$ subjects, 20 are classified as $L3b$ and 15 as $L3a$ (the remaining 4 subjects are classified as $L0$ or the unidentified). The results show that subjects' belief levels and ability are mostly consistent throughout the experiment, and that about half of the subjects using two or three steps have reached their upper boundaries of reasoning.

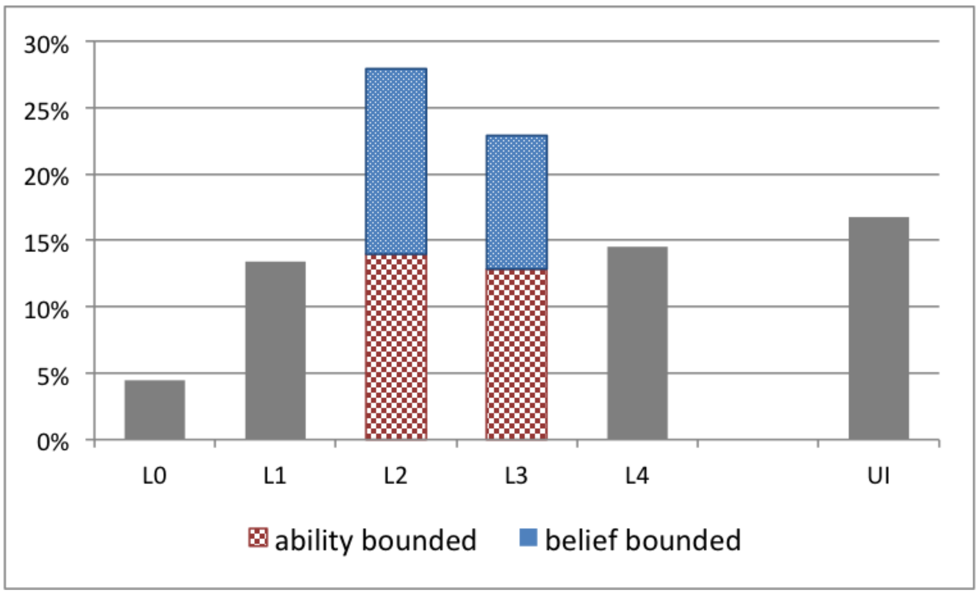


Figure 2: Decomposition of $L2$ and $L3$ subjects

I further investigate whether the subjects' belief is based on their opponents' belief or ability. This is tested on the $L3$ subjects. In both sets of the sequential games, $L3$ players need to think four steps. They differ in that, in order to play the iterative dominant strategies as player 1, in SEQ-P3 players need to believe that the opponents are $L2b$, while in SEQ-P2 they could believe that the opponents are $L2a$ or $L2b$. The performances of the 39 $L3$ subjects are not statistically different in the two sets of games. In both SEQ-P2 and SEQ-P3, there are 16 out of these 39 subjects that best respond at player 1 positions, which supports the hypothesis that the $L3b$ players believe that their opponents are $L2b$.

To help better understand the correlation between cognitive ability and observed levels, the Cognitive Reflection Test (CRT) scores were collected from the participants after the main part of the experiment. The CRT score is a measure of cognitive ability in decision making. I find that the CRT scores increase with levels among the lower types, and then flatten out. Especially, I find that $L2b$ subjects performed much better in CRT than $L2a$. But among the players who use at least three steps of reasoning in the ring games, the differences in CRT scores are not significant.

Overall, I find the existence of both the high ability subjects responding to lower-order belief and the low ability subjects who could only think two or three steps, which shows large heterogeneity in subjects' reasoning ability. The results suggest that both subjects' belief and their reasoning ability could be the decisive factor of their exhibited levels. The CRT scores support the separation of $L2b$ and $L2a$ subjects.

The remainder of the paper proceeds as follows. The next section summarizes the related literature. The experimental design and detailed identification strategies are presented in Section 3. Section 4 reports the experimental results. Section 5 concludes.

2 Related Literature

The recent two decades saw the emergence of a vast literature on the level-k model. A detailed review is given by Crawford et al. (2013). This section summarizes only the studies that are most closest to this paper. The simultaneous ring games used in this study are similar to the ones proposed by Kneeland (2015). Her methodology and results will be discussed in detail in the following sections, and hence are not included here. There is also a class of literature assuming that Lk players believe that the opponents are drawn from a distribution of all the lower types, e.g. the Cognitive hierarchy model from Camerer et al. (2004). Since the games in this study do not distinguish between the level-k model and the Cognitive Hierarchy model, players in this study are assumed here to have degenerate belief, as in the original level-k model.

There are a few studies looking at subjects' reaction to the information on the opponents' types. Given the information that the opponents are higher types, the high ability Lkb subjects should be able to respond by raising their own levels, but the ability-bounded Lka ones are not able to adjust their behavior. Some studies find positive results, suggesting that the participants are Lkb. For example, Palacios-Huerta and Volji (2009) invite chess players, presumably the higher types, and college students to play the centipede game, and find that they both exhibit higher levels when playing against chess players than against students. Agranov et al. (2012) find that in a 2/3 beauty-contest game, subjects exhibit higher sophistication levels (lower average choice numbers) as the number of experienced players, some graduate students in economics, increases in the group. In Slonim (2005) experienced subjects are found to respond to the experience levels of their opponents. Alaoui and Panta (2015) also find that subjects respond to the manipulation of their beliefs.

In some other studies, the results are mixed. For example, in Gill and Prowse (2015), who use repeated beauty contest games, the higher cognitive ability subjects respond positively to the cognitive abilities of their opponents, while the lower cognitive ability subjects do not. Their findings imply that only high types have the ability to respond. In addition, Georganas et al. (2015) find such effect in some games, but not in others. They ask the subjects to choose the strategies for the opponents randomly selected from the whole sample, the higher cognitive ability half and the lower half, and find significantly more higher levels in the responses to higher types in the undercutting games, but no such effect in the two-people guessing games.

A more direct way to identify whether subjects are best responding to their belief is to elicit their belief. Belief elicitation in strategic games has been studied in numerous papers. However, evidence is mixed on whether beliefs could be successfully elicited without altering behavior and whether subjects do act according to their stated beliefs. Some studies find that subjects' actual play in the games is not affected after belief elicitation (Nyarko and Schotter (2002), Costa-Gomes and Weizsacker (2008), Manski and Neri (2013)), while both Ruststrom and Wilcox (2009) and Gachter and Renner (2010) find that incentivized elicitation alters choices. In both Nyarko and Schotter (2002) and Manski and Neri (2013), most subjects appear to be best responding to their first-order belief. However, in Bhatt and Camerer (2005), only 66% of the choices match with the stated first-order belief. Costa-Gomes and Weizsacker (2008) also find that most choices are one step below the stated first-order belief.

Additional strategies are developed in order to show that high ability subjects are responding to low-order belief. Costa-Gomes et al. (2001) and Costa-Gomes and Crawford (2006) train subjects to play against robots programed to use certain Lk or equilibrium decision rules. They find that the subjects have no problem responding to any of these rules, which implies that they have the enough reasoning ability to reach equilibrium. Arad and Rubinstein (2012) use a very simple undercutting game, but still find that subjects mostly use no more than three steps of reasoning. They conclude that this could not be due to obstacles in thinking, and thus must be due to non-equilibrium beliefs. Other methods of identifying belief and strategic thinking processes include tracking players' information search (Costa-Gomes et al. (2001), Costa-Gomes and Crawford (2006) and Brocas et al. (2014)), or translating their beliefs from communication records between players (Burchardi and Penczynski, 2014). Bhatt and Camerer (2005) use brain image to search for the connection between the brain activities of making choices and stating belief.

The mixed findings in the previous studies suggest heterogeneity in the population. The results in this paper, that there exist both belief-bounded and ability-bounded subjects, are in line with the literature. Since the identification in this paper only requires choice data, it circumvents the challenge of belief manipulation or belief elicitation, which makes it easier to achieve a within-subjects design.

In an ongoing project, Friedenberg, Kets and Kneeland (2016) also studies the relationship between belief and reasoning abilities in a ring game setting. They look at whether a subject classified as Lk uses the information more than k steps away in a pair of simultaneous ring games. Although our works differ in identification strategies and the definitions of reasoning abilities, their main finding that people's exhibited levels do not always reflect their true abilities is consistent with the results in this paper.

Besides which, this paper finds positive correlation between CRT scores and observed levels, which extends the literature that explores the relationship between cognitive ability and strategic thinking. One of the closely related studies is Gill and Prowse (2015), who use the Raven's test as a measure of cognitive ability and find that the high ability subjects converge to equilibrium more quickly and earn more. Another related study, Georganas et al. (2015), uses four tests, including the CRT. They find that none of the test scores is a good prediction of observed levels, but CRT scores could predict earnings. In addition, Rydval et al. (2009) find that higher working memory, need for cognition, and premeditation are associated with a higher likelihood of obeying dominance. Works on the Beauty Contest Games also show that the subjects who score high in the CRT or other tests cognitive ability play the strategies closer to equilibrium (Burnham et al., 2009; Schnusenberg and Gallo, 2011; Branas-Garza et al., 2012).

This paper is also related to the literature that studies level-k models in sequential games. For examples, see the two stage games in Stahl and Haruvy (2008), and the club game in Breitmoser et al. (2014). Ho and Su (2013) provide a dynamic level-k model to study the centipede games. To the best of my knowledge, the design in this paper is the first to create more than k reasoning steps for a Lk belief using sequential moves.

3 Experimental Design and Identification Strategy

3.1 Some Notations

In the dominance-solvable ring games, $L0$ is assumed to be unable to identify a strictly dominated strategy. $L1$ best responds to the belief that others do not obey strict dominance. $L2$ best responds to the belief that others are $L1$. For each $k \geq 2$, Lk holds the belief that the opponents are $L(k-1)$ and best respond accordingly.

In previous studies, a player is classified as Lk if he exhibits Lk behavior. That is, he plays best responses to the Lk type's belief. This study distinguishes between the ones who hold Lk belief and have the ability to best respond, and the ones who might believe that others are higher than $L(k-1)$, but behave like Lk because they could not identify the best responses to higher-order beliefs.

I assume that each player is characterized by his belief level b and his ability level a ($b, a = 1, 2, 3, ...$). Similar setup could be found in Strzalecki (2014), Alaoui and Panta (2015) and Georganas et al. (2015). Here $b = k$ if the player holds Lk type's belief, and $a = k$ if the player is able to do at most k steps of iterative thinking. The belief level b and ability level a are not directly observable from the choice data. Since in most games used in previous studies, it requires k steps of reasoning to respond to Lk belief. Therefore when a player chooses a best response to Lk belief, it could only be inferred that he has $\min\{a, b\} = k$.

In this study I try to separate the two cases. If a player exhibits Lk behavior due to his belief and has much higher ability, he is called Lkb and has $b = k$ and $a > k$. In the other case, a player behaves as Lk because he could think at most k steps in this game. He is called Lka and has $b \geq k$ and $a = k$. Non-strict inequality is used here, because it is indistinguishable whether his belief level is also k or he has higher level belief but is not able to best respond.

Note that here b is defined by the belief in others' belief. A player has $b = k$ if he believes that others best respond to $b = k - 1$. In this experiment, I also test whether there exist players who hold the belief in others' ability (that others have $a = k-1$) instead of others' belief.

3.2 The Games

The main part of the experiment consists of three sets of 4-player ring games, which are called SIMUL (the simultaneous games[1]), SEQ-P2 and SEQ-P3 (the sequential games) (see Figure 3). The SIMUL set contains two ring games, G1 and G2, with simultaneous moves. The two sets of sequential games are labeled by the second movers. In the games of SEQ-P2, G3 and G4, player 2 is the second mover. Players 1, 3 and 4 move simultaneously in the first stage and player 2 moves in the second stage after observing their actions. In the SEQ-P3 games, G5 and G6, player 3 moves in the second stage. The sequence of move and the information structure are common knowledge among all the players.

Each player has three actions to choose from, and the payoffs are represented by 3×3

[1]The payoffs in these games differ from the original ring games in Kneeland (2015) in two ways. Firstly, the row with the largest sum in each of the first three matrices is never an iterative dominant strategy. It was shown in Jin (2016) that the rows with the largest sums are usually the first choices of the non-strategic subjects. If these rows also happen to be the iterative dominant strategies, subjects' types could be overestimated. In addition, none of the rows contains the salient number 0. Jin (2015) provides the evidence that the 0s as possible minimum payoff could divert the players from choosing these actions and bias type classification. Thus it would help get a better separation of the Lk types by taking care of these two issues.

matrices. Player 1's (P1) payoff is determined jointly by his own action and his direct opponent player 2's (P2) action. Player 2's payoff is determined by his and player 3's (P3) actions. Player 3's payoff depends on his and player 4's (P4) actions. Player 4's payoff depends on his and player 1's actions.

In these ring games, player 4 always has strictly dominant strategies. Therefore in each game there is a unique Nash equilibrium, which could be solved from four rounds of iterated elimination of strictly dominated strategies.

In addition, in each pair of rings (G1 and G2, G3 and G4, G5 and G6), the payoff matrices of the two players 1, 2 and 3 positions are identical. The two player 4 positions have the same three strategies, but the strategies are labeled in different sequences. This feature, combined with the dominant strategies at player 4 positions, is designed specifically for type classification, which will be discussed in the next subsection.

3.3 Lk Types Classification

Subjects' Lk behavior could be inferred from their choices in the simultaneous games. The identifying assumption **ER** (the *exclusive restriction* in Kneeland (2015)) is applied to separate subjects with different levels. The assumption ER says that a player with Lk but not higher order belief does not respond to the changes in $(k+1)$th- or higher-order payoffs. It is weaker than the assumption that $L0$ chooses all the possible actions with equal probability. The latter is a special case of ER. The type classification using the uniformly randomizing $L0$ assumption appears to be less robust among lower types, but the pattern in the main result holds for higher types. These results will be reported in the appendix.

A player's kth-order payoff is defined as his kth-order belief about payoffs. Therefore a player's 1st-order payoff is his own payoff. In the simultaneous ring games, each player only forms belief on his direct opponent. Therefore the player's 2nd-order payoff is his opponent's payoff, and his 3rd-order payoff is the payoff of his opponent's opponent, and so on.

ER could be used to model players' off-equilibrium behavior in ring games[2]. It implies that a low type player's choice will not be affected by the changes in the other players' decision situations, if the changes do not look so relevant to his own payoff.

In G1 and G2, the payoff matrices are identical in the players 1, 2 and 3 positions

[2]In most other games without ring structure, the second-order opponent is usually the player himself. Hence this identification strategy can be used to identify up to $L2$ players in these games. In ring games, by increasing the number of players in a ring, a player's high-order opponent's payoff functions could be perturbed without affecting his own payoff and lower-order payoffs. Thus ER could be used to separate higher-orders of rationality as long as enough players are included in the rings.

Player 1 \ Player 2	$a2$	$b2$	$c2$
$a1$	30	6	20
$b1$	4	20	32
$c1$	12	40	6

P1's Payoff

Player 2 \ Player 3	$a3$	$b3$	$c3$
$a2$	32	24	4
$b2$	36	20	10
$c2$	26	18	16

P2's Payoff

Player 3 \ Player 4	$a4$	$b4$	$c4$
$a3$	18	20	26
$b3$	14	8	36
$c3$	6	24	28

P3's Payoff

Player 4 \ Player 1	$a1$	$b1$	$c1$
$a4$	8	12	24
$b4$	20	16	32
$c4$	18	14	20

P4's Payoff

SIMUL: G1

Player 1 \ Player 2	$a2$	$b2$	$c2$
$a1$	30	6	20
$b1$	4	20	32
$c1$	12	40	6

P1's Payoff

Player 2 \ Player 3	$a3$	$b3$	$c3$
$a2$	32	24	4
$b2$	36	20	10
$c2$	26	18	16

P2's Payoff

Player 3 \ Player 4	$a4$	$b4$	$c4$
$a3$	18	20	26
$b3$	14	8	36
$c3$	6	24	28

P3's Payoff

Player 4 \ Player 1	$a1$	$b1$	$c1$
$a4$	18	14	20
$b4$	8	12	24
$c4$	20	16	32

P4's Payoff

SIMUL: G2

Player 1 \ Player 2*	$a2$	$b2$	$c2$
$a1$	16	32	8
$b1$	28	16	14
$c1$	14	10	32

P1's Payoff

Player 2* \ Player 3	$a3$	$b3$	$c3$
$a2$	6	30	28
$b2$	22	28	8
$c2$	4	20	36

P2*'s Payoff

Player 3 \ Player 4	$a4$	$b4$	$c4$
$a3$	22	8	24
$b3$	28	12	20
$c3$	32	4	18

P3's Payoff

Player 4 \ Player 1	$a1$	$b1$	$c1$
$a4$	24	20	32
$b4$	16	12	28
$c4$	22	18	8

P4's Payoff

SEQ-P2: G3

Player 1 \ Player 2*	$a2$	$b2$	$c2$
$a1$	16	32	8
$b1$	28	16	14
$c1$	14	10	32

P1's Payoff

Player 2* \ Player 3	$a3$	$b3$	$c3$
$a2$	6	30	28
$b2$	22	28	8
$c2$	4	20	36

P2*'s Payoff

Player 3 \ Player 4	$a4$	$b4$	$c4$
$a3$	22	8	24
$b3$	28	12	20
$c3$	32	4	18

P3's Payoff

Player 4 \ Player 1	$a1$	$b1$	$c1$
$a4$	16	12	28
$b4$	22	18	8
$c4$	24	20	32

P4's Payoff

SEQ-P2: G4

Player 1 \ Player 2	$a2$	$b2$	$c2$
$a1$	18	30	10
$b1$	36	8	14
$c1$	20	14	34

P1's Payoff

Player 2 \ Player 3*	$a3$	$b3$	$c3$
$a2$	20	10	26
$b2$	34	8	14
$c2$	16	32	10

P2's Payoff

Player 3* \ Player 4	$a4$	$b4$	$c4$
$a3$	8	18	36
$b3$	28	32	6
$c3$	16	40	8

P3*'s Payoff

Player 4 \ Player 1	$a1$	$b1$	$c1$
$a4$	8	24	16
$b4$	24	28	40
$c4$	18	6	32

P4's Payoff

SEQ-P3: G5

Player 1 \ Player 2	$a2$	$b2$	$c2$
$a1$	18	30	10
$b1$	36	8	14
$c1$	20	14	34

P1's Payoff

Player 2 \ Player 3*	$a3$	$b3$	$c3$
$a2$	20	10	26
$b2$	34	8	14
$c2$	16	32	10

P2's Payoff

Player 3* \ Player 4	$a4$	$b4$	$c4$
$a3$	8	18	36
$b3$	28	32	6
$c3$	16	40	8

P3*'s Payoff

Player 4 \ Player 1	$a1$	$b1$	$c1$
$a4$	18	6	32
$b4$	8	24	16
$c4$	24	28	40

P4's Payoff

SEQ-P3: G6

Note: * denotes second movers.

Figure 3: The ring games

respectively. The only difference is in the payoff matrices of player 4. Hence ER implies that an $L1$ subject chooses the dominant strategies as player 4 but makes the same choice in both rings as players 1, 2 and 3. The behavior pattern of other Lk types could also be predicted. An $L2$ subject makes the same choices as players 1 and 2 in both G1 and G2, and chooses the (iterative) dominant strategies as players 3 and 4. An $L3$ subject makes the same choice at the two player 1 positions and chooses the (iterative) dominant strategies as players 2, 3 and 4. $L4$ or higher types choose the (iterative) dominant strategies at all player positions.

3.4 Separation of Lkb and Lka

Since in the simultaneous games, responding to Lk belief requires k steps of reasoning, Lkb and Lka subjects should behave like the same Lk type. Separation could be done through comparing their behavior between simultaneous and sequential games. But in order to make meaningful comparisons, it is essential to assume that a subject's belief level and ability level remain constant in these games.

A1: As a first mover, a subject's upper boundary of reasoning steps stays the same in simultaneous and sequential ring games.

A2: A subject's belief about first mover opponents stays the same in simultaneous and sequential ring games.

Assuming that a subject's type remains constant over similar games is standard in previous studies that involve within-subjects design (Stahl and Wilson, 1995; Costa-Gomes et al., 2006; Costa-Gomes and Crawford, 2006). In this study, I need to additionally assume that adding sequential moves does not change a subject's belief. This assumption is crucial, for one's belief and reasoning ability are likely to vary with the complexity of the games (Georganas et al., 2015). I would like to argue that this is not the case for the first movers.

Although the games in SEQ-P2 and SEQ-P3 involve sequential moves, they share the same payoff structure and reasoning process with the simultaneous games. Thus, as a first mover, one needs to go through the same thought process to obey iterative dominance in both the simultaneous and sequential games. So it is assumed in A1 that a subject's ability level should not change as a first mover.

Due to the same reason, a subject's belief in the first movers' rationality should not change either. For example, if a subject believes in the simultaneous games that his opponent is capable of choosing the strictly dominant action from a 3×3 matrix, he should hold the same belief in the sequential games on first movers. Similarly, the higher-order beliefs on first movers also stay the same.

As for the second movers, although it also takes $L1$ belief and one step of reasoning to best respond, the task complexity is not exactly the same as the $L1$ task in the simultaneous game. Thus, I only assume that the task for the second mover is easier, if not the same, than the task for player 4 in the simultaneous games.

A3: If a subject believes that the opponents obey strict dominance in the simultaneous games, then he believes that they are capable of best responding as the second movers in the sequential games.

With these three assumptions, behavioral patterns of Lkb and Lka subjects in the sequential games could be predicted, as shown in Table 2.

		P1	P2	P3	P4			P1	P2	P3	P4
$L1b$	SEQ-P2	×	$2nd$	×	√	$L1a$	SEQ-P2	×	$2nd$	×	√
	SEQ-P3	×	×	$2nd$	√		SEQ-P3	×	×	$2nd$	√
$L2b$	SEQ-P2	×	$2nd$	√	√	$L2a$	SEQ-P2	×	$2nd$	√	√
	SEQ-P3	×	√	$2nd$	√		SEQ-P3	×	×	$2nd$	√
$L3b$	SEQ-P2	√	$2nd$	√	√	$L3a$	SEQ-P2	×	$2nd$	√	√
	SEQ-P3	√	√	$2nd$	√		SEQ-P3	×	√	$2nd$	√
$L4b$	SEQ-P2	√	$2nd$	√	√	$L4a$	SEQ-P2	√	$2nd$	√	√
	SEQ-P3	√	√	$2nd$	√		SEQ-P3	√	√	$2nd$	√

Note: √ denotes choosing the (iterative) dominant strategy at this position, × otherwise. "2nd" denotes second movers. All the types in this table should play the dominant strategies in the subgame as second movers.

Table 2: Predicted action profiles of each Lkb and Lka type

$L2b$ and $L2a$ could be separated using the player 2 positions of SEQ-P3. In SEQ-P3, if player 2 has at least $L2$ belief, he would believe that player 4 chooses the dominant strategy. In addition, given A3, he should also believe that player 3 best responds after observing player 4's action. Therefore, it requires $L2$ belief and three steps of reasoning to play the iterative dominant strategies at the player 2 positions of SEQ-P3. An $L2b$ subject ($b = 2, a > 2$) will be able to solve for the iterative dominant strategies at these positions, while an $L2a$ subject ($b \geq 2, a = 2$) is not able to think three steps and thus not able to best respond.

Similarly, $L3b$ and $L3a$ could be separated using the player 1 position of SEQ-P2 and SEQ-P3. In SEQ-P2, a player 1 with at least $L3$ belief would believe that player 3 chooses the iterative dominant strategy, and that the second mover, player 2, best responds to his observation of player 3's action. In SEQ-P3, a player 1 with at least $L3$ belief should also believe that the opponent best responds as player 2, which requires $L2$ belief. Therefore, it requires $L3$ belief and four steps of reasoning to play the iterative dominant strategies at the player 1 positions in both SEQ-P2 and SEQ-P3. So the $L3b$ subjects ($b = 3, a > 3$) are able to best respond as player 1, but the $L3a$ subjects ($b \geq 3, a = 3$) are not.

In addition, by comparing $L3$ subjects' behavior in SEQ-P3 and SEQ-P2, it could be tested whether $L3b$ subjects believe that their opponents hold $L2$ beliefs or they believe that their opponents are only able to think two steps. In SEQ-P2, $L2b$ or $L2a$ are not distinguishable. A subject classified as $L3b$ might believe that the opponents are $L2b$ or $L2a$. But in SEQ-P3, $L3b$ needs to believe that the opponents are $L2b$. Therefore if there exists any subject who believes that the opponents are $L2a$, he will be classified as $L3b$ in SEQ-P3, but $L3a$ in SEQ-P2.

With this design, $L1b$ and $L1a$ cannot be separated. $L4b$ and $L4a$ could be separated with 5-player ring games, which are not used in this experiment.

3.5 Experimental Procedures

The experiment was conducted at the Experimental Social Science Laboratory (XLab) at UC Berkeley. A total of 184 subjects, who were recruited from Berkeley undergraduate classes, participated in 6 different sessions, with between 20 to 36 subjects in each session. Each experimental session lasted about one and a half hours. The average earnings were \$25, plus a \$5 participation fee, which were paid in private after each session.

The data was collected through an online interface. Subjects were not allowed to interact directly, and their identities were kept confidential. After subjects read the instructions, the instructions were read aloud by an experimenter. Then the subjects were given a short quiz to test their understanding of the game structure, followed by 4 unpaid practice games to help them get familiar with the interface.

In the main part of the experiment, the subjects played at the 24 positions of the games in a random order. In each game they were matched with a new group. Though the subjects were not allowed to write during the experiment, our online interface allowed them to mark any cell in the payoff matrices by a click of the mouse. In this way they were able to easily track the equilibrium strategies across matrices. The subjects were not allowed to make changes once they had confirmed their choices.

There was a time limit of 60 seconds for the subjects to complete each game. If they failed to choose in one game, the earnings for this game would be zero, and the system would randomly pick from the three choices for them to calculate the payoffs of their opponents. The second movers were given an additional 30 seconds after the first movers had submitted their choices[3]. The decision time of each game was recorded for each subject.

After the subjects had completed all 24 games, they took the Cognitive Reflection Test. The CRT is composed of three short questions and is designed to measure subjects' cognitive ability (Frederick, 2005).

At the end of each session, subjects finished a survey of their demographic characters. One of the 24 games was randomly chosen for payment. One out of the three questions in the CRT were chosen for payoff and the subjects got \$0.25 if their answers were correct.

4 Experimental Results

This section starts by reporting the descriptive statistics and the identification of *Lk* behavior from subjects' choices in the simultaneous games. Then I analyze the behavioral patterns of each type in the sequential games, and separate the *Lkb* and *Lka* subjects. In addition, the CRT scores are used to study the correlation of subjects' identified types and a measure of their cognitive ability. A robustness test of learning effects is provided at the end of the section.

4.1 Data Description

The percentage of subjects who choose the (iterative) dominant strategies at each of the 24 positions is reported in Table 3. For the second movers, I look at whether they choose the dominant strategies in the subgames. Out of the 184 participants, 172 finished all the choices within the time limit; 7 failed to choose in one game but the missing choices do not affect their type classification. The following analysis is based on the 179 subjects. The 5 subjects are excluded because they failed to choose in one game and the missing choices affect their type classification.

The compliance rates are quite high at all of the player 4 positions and the second mover positions (player 2 of SEQ-P2 and player 3 of SEQ-P3). Over 95% of the subjects choose the dominant strategies at these positions, which suggests that the majority of the participants understand the payoff structure and are capable of best responding to strict dominance. The compliance rates of iterative dominance decrease as the required reasoning steps go up. In the simultaneous games, compliance rates are the highest at player 4 positions, followed by players 3, 2 and 1. Similar patterns are observed in the sequential games. This is consistent with the prediction of the level-k model in that fewer

[3]There are 12 subjects who failed to choose in one game (out of 184 $\times$ 24 games in total). For 7 out of the 12 subjects, missing one choice does not affect their type identification. Five subjects (406, 412, 610, 814, 1017) are affected and are excluded from the type distribution presented in the following section.

players achieve higher rounds of iterative thinking.

A trace of of the treatment effects of the sequential moves could be found in the differences in compliance rates. For example, player 2 positions of both SIMUL and SEQ-P3 require three steps of reasoning, but the player 2 position requires only $L2$ belief in SIMUL and $L3$ belief in SEQ-P3. If subjects' behavior is solely determined by their beliefs, higher compliance rates are expected at the player 2 position of SEQ-P3, as more people have 2nd-order belief in rationality compared to 3rd-order belief. Such a pattern could be found in the data. However, the compliance rates at player 2 positions of SEQ-P3 are lower than player 3 of SIMUL, which also requires $L2$ belief. It might imply that not every subject who exhibits $L2$ behavior in the simultaneous games is able to proceed to three steps of thinking. A similar pattern is found for $L3$ subjects. That is, the compliance rates of the player 1 positions in SEQ-P2 and SEQ-P3 are higher than the player 1 positions in SIMUL, but lower than player 2 of SIMUL. Of course, it requires further investigation to tell whether these changes do come from the $L2$ and $L3$ subjects.

		player 1	player 2	player 3	player 4
SIMUL	G1	34.2%	56.0%	77.7%	97.3%
	G2	46.7%	54.3%	79.3%	97.8%
		player 1	player 2 (2nd)	player 3	player 4
SEQ-P2	G3	47.3%	95.1%	80.4%	96.7%
	G4	42.9%	93.5%	77.7%	94.6%
		player 1	player 2	player 3 (2nd)	player 4
SEQ-P3	G5	49.5%	71.2%	96.2%	98.4%
	G6	51.6%	65.2%	96.2%	97.3%

Note: $N = 179$.(2nd) denotes second stage movers.

Table 3: Compliance rates of (iterative) dominating strategies

4.2 Observed Lk Behavior in Simultaneous Games

A subject would exhibit Lk behavior in the simultaneous games if he holds at least Lk belief and has the ability to perform at least k steps of reasoning. Up to $L4$ behavior could be identified from subjects' choices in the simultaneous games.

The type classification method is from Kneeland (2015). It is assumed that each subject's behavior is determined by a single type, which remains constant throughout the experiment. A subject i deviates from his Lk type's choice profile with probability ϵ_{ik}, which is i.i.d. across games. When a subject deviates from his own type, it is assumed that he chooses the other two strategies with equal probability. The likelihood of a player i being type k given his action profile can be defined as

$$d_{ik}(\epsilon_{ik}, x_{ik}) = (1-\epsilon_{ik})^{G-x_{ik}}(\frac{\epsilon_{ik}}{2})^{x_{ik}}, \tag{1}$$

where G is the number of games and x_{ik} is the number of observations that do not match the predicted profile of type k.

A subject is assigned to the type k with the highest likelihood d_{ik}, which is equivalent to finding the lowest number of deviations x_{ik}. If a subject's action profile matches exactly with a type's predicted profile, he will be assigned to this type. However, if $\min_k(x_{ik}) > 0$, there might be more than one minimum x_{ik}. Following Kneeland (2015), this subject will be assigned to the lowest type that has the minimum number of deviations.

If an action profile deviates too much from the predicted profiles of $L1$-$L4$, he would be labeled as the irrational $L0$ or the *unidentifiable* type. The unidentifiable subjects are defined as deviating from Lk predictions, but picking the dominant strategies as player 4 in both two rings. Hence they are at least capable of best responding and should be distinguished from the irrational, unpredictable $L0$ type. The unidentified types might be using some rules of their own that cannot be captured by our model.

Therefore a cutoff point is needed so that those subjects with $\min_k(x_{ik})$ larger than the cutoff will be assigned to $L0$ or the unidentifiable. Table 4 reports the type assignment results with the cutoffs being 0, 1 or 2 deviations. In the first row, when a subject cannot be matched exactly to a Lk type, he is assigned to $L0$ or unidentifiable. This seems to be too strict as there are over 60% of the subjects left unidentified. When allowing for 1 deviation, the share of the unidentified subjects drops down to 15%, with more subjects being assigned to one of the four Lk types. If the 2-deviation cutoff is used, there is a further drop in the number of the unidentified subjects and an increase in $L1$ and $L2$. The numbers of higher types do not change.

To determine which cutoff is the most appropriate in this study, a sample of 10,000 random choosing subjects is simulated and analyzed through the type assignment process. The analysis focuses on how many of these subjects could be correctly assigned to $L0$ and how many are wrongly assigned to one of the Lk types. When allowing for 1 deviation, over 86% of them are classified as $L0$, and around 5% go to the Lk types. It does not differ too much from the 0-deviation cutoff. However, when allowing for 2 deviations, over 25% of the random choosing subjects are assigned as Lk, which is too high to be acceptable. So

Level	$L0$	$L1$	$L2$	$L3$	$L4$	UI
0 deviation	8	5	17	17	19	113
	4.5 %	2.8 %	9.5 %	9.5 %	10.6 %	63.1 %
random choice	89.0 %	0.4 %	0.1 %	0.1 %	0.0 %	10.4 %
1 deviation	7	26	50	39	30	27
	3.9 %	14.5 %	27.9 %	21.8 %	16.8 %	15.1 %
random choice	86.4 %	4.4 %	1.0 %	0.3 %	0.2 %	7.7 %
2 deviations	5	40	56	39	30	9
	2.8 %	22.3 %	31.3 %	21.8 %	16.8 %	5.0 %
random choice	72.5 %	18.9 %	4.2 %	1.5 %	0.7 %	2.2 %

Note: $N = 179$. Each subject is classified as $L1 - L4$ with no more than 0, 1 or 2 deviations from the predicted action profiles. Otherwise they are assigned to $L0$ or unidentifiable. The subjects classified as unidentifiable are able to choose dominant strategies as Player 4 but do not match any of the predicted patterns. They are at least rational, which makes them different from $L0$. The random choices are simulated with 10,000 randomly choosing subjects.

Table 4: Type assignment from SIMUL games

it appears that the 1-deviation cutoff is the most appropriate, for it gives reliable results and provides enough observations of the Lk types for the following analysis. The results using the 0-deviation and 2-deviation cutoffs, from which a similar pattern could be found as in the main results, are reported in the appendix.

The type classification in Kneeland (2015) also uses the 1-deviation cutoff. The 1-deviation cutoff type distribution in my data is very close to the distribution found in Kneeland (2015). If the unidentified subjects are excluded, the Fisher's exact test comparing these two categorical distributions yields a p-value of 0.926, suggesting that they are statistically not different. However, there is a much larger proportion of unidentifiable of 15.1%, compared to 1.2% in Kneeland's (2015) data[4], which is due to larger deviation rates of my subjects. It should be noted that even with the 2-deviation cutoff and thus only 5% unidentifiable subjects, Fisher's exact test still does not reject that the distribution in this study is different from Kneeland's (2015), though with a lower p-value of 0.853.

Just as the one in Kneeland (2015), this type distribution is relatively higher than the literature (for example, Stahl and Wilson (1995) and Costa-Gomes et al. (2001)), which might be atributed to the special features of ring games. There are 27.9% and 21.8% of

[4]Kneeland (2015) does not include a category of unidentifiable subjects, since there is only one such subject in her main treatment. This subject is assigned to $L0$ in her original paper.

the subjects classified as $L2$ and $L3$ respectively with the 1-deviation cutoff, which serves as the starting point of the following analysis on the separation of Lkb and Lka.

4.3 Behavioral Pattern of Each Lk Type in the Sequential Games

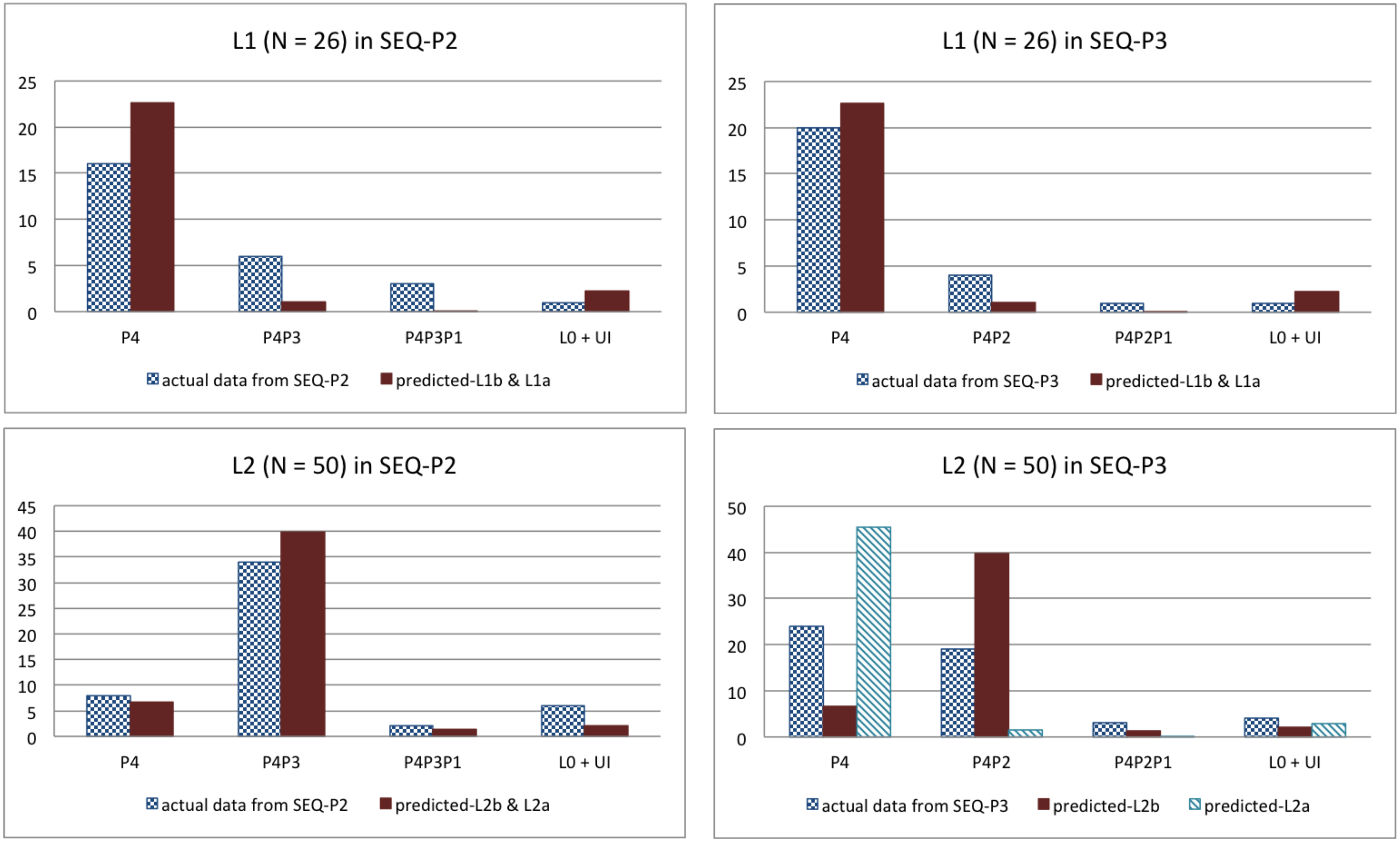
In this section, the behavioral pattern of each Lk type, especially $L2$ and $L3$, is analyzed to determine the existence of Lkb and Lka subjects. I first show that most subjects' behavior falls into the predicted categories, which supports the consistency of subjects' belief levels and ability levels across three sets of games. I then provide evidence on the existence of both Lkb and Lka subjects by a closer look at the patterns of the $L2$ and $L3$ subjects. Finally, I show that the $L3b$ subjects hold the belief that their opponents are $L2b$, by comparing the $L3$ subjects' behavior in the two sets of sequential games.

Result 1 Subjects' behavioral patterns in the sequential games are close to the theoretical predictions assuming that their belief levels and ability levels are consistent across simultaneous and sequential games. Most subjects fall into the predicted categories.

The dark bars of Figure 4 and Figure 5 describes the behavioral pattern of each type in the sequential games. According to Table 2, subjects' behavior could be sorted into three categories based on their choices as first movers. In SEQ-P2, the subjects could be playing the (iterative) dominant strategies only at P4 position, at P4 and P3 positions, or at all of the P4, P3 and P1 positions. In SEQ-P2, the three categories are playing the (iterative) dominant strategies at P4 position, at P4 and P2 positions, or at P4, P2 and P1 positions. The three categories correspond to the subjects who are still best responding to $L1$, $L2$ and $L3$ belief in the sequential games. Each subject could be assigned to one of the categories by the method in Section 4.2 and the 1-deviation cutoff[5]. If a subject deviates too much from these categories, e.g. best responds as player 1 but not as player 4, it means that either he is irrational or his behavior could not be explained by the theoretical model. Such a subject will be assigned to $L0$ or remain unidentified if he chooses the dominant strategies as player 4.

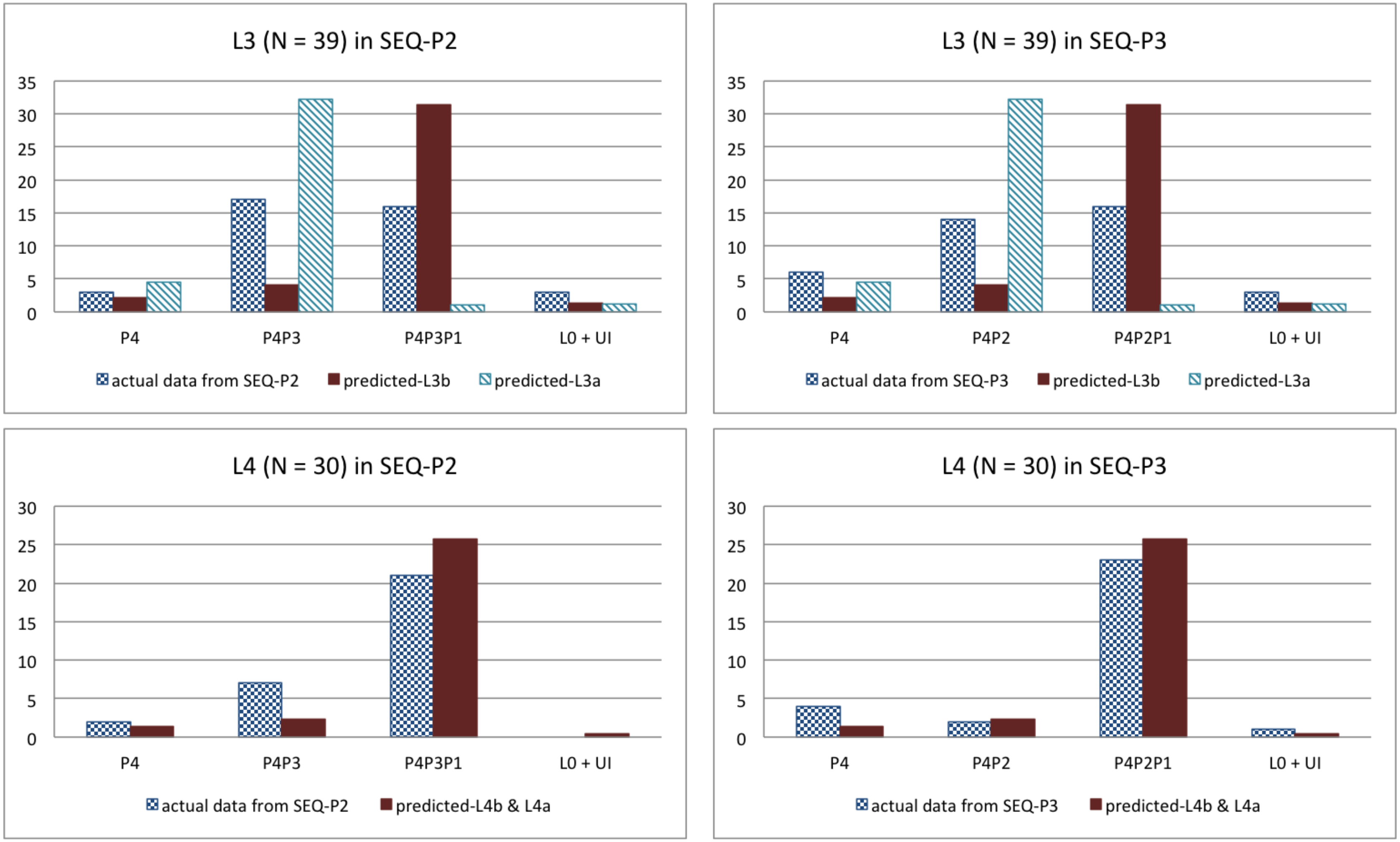
In theory, these subjects' behavior should follow Table 2 if their belief levels and ability levels remain the same in both simultaneous and sequential games. However, since subjects sometimes deviate from their own types, one could be misidentified if he has too many

[5]According to ER, a subject best responding to $L1$ belief in SEQ-P2 plays dominant strategies as player 4, but chooses the same actions as players 1 and 3. This gives the predicted action profile for the "P4" category in SEQ-P2, which includes $L1b$ and $L1a$. Similarly, a subject responding to $L2$ belief in SEQ-P2 should play the (iterative) dominant strategies as players 3 and 4 and choose the same action as player 1, which gives the action profile for the "P4P3" category in SEQ-P2, including $L2a$, $L2b$ and $L3a$. A subject responding to $L3$ belief in SEQ-P2 should play the (iterative) dominant strategies as players 1, 3 and 4, which is the "P4P3P1", including $L3b$, $L4a$ and $L4b$. The actions of the three categories in SEQ-P3 could be predicted in the same way.



Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P3 (“P4P3”), and at P4, P3 and P1 (“P4P3P1”). In SEQ-P3, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P2 (“P4P2”), and at P4, P2 and P1 (“P4P2P1”). Each subject is assigned to a category with ER and 1-deviation cutoff. The dark blue bars give the actual type distribution. The light blue and medium blue bars give the simulated type distribution assuming that these subjects are Lkb or Lka. In certain cases the behaviors of Lkb and Lka are not distinguishable, and they are represented by the same light blue bar.

Figure 4: Behavior pattern of each Lk type in sequential games



Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 ("P4"), at P4 and P3 ("P4P3"), and at P4, P3 and P1 ("P4P3P1"). In SEQ-P3, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 ("P4"), at P4 and P2 ("P4P2"), and at P4, P2 and P1 ("P4P2P1"). Each subject is assigned to a category with ER and 1-deviation cutoff. The dark blue bars give the actual type distribution. The light blue and medium blue bars give the simulated type distribution assuming that these subjects are Lkb or Lka. In certain cases the behaviors of Lkb and Lka are not distinguishable, and they are represented by the same light blue bar.

Figure 5: Behavior pattern of each Lk type in sequential games (Cont'd)

deviations. In order to determine how many deviations from the predicted categories could occur due to misidentification, for each of the $L1$-$L4$ types, I simulate the choices of 10,000 pseudo-subjects in sequential games, assuming that all of them are either Lkb or Lka. The average deviation rate of each type used in the simulation is from the type classification results of the simultaneous games in Section 4.2. In Figure 4, the lighter bars of each category give the predicted behavior pattern of Lkb or Lka obtained from simulation. According to the simulation, around 85% of the subjects should fall into the predicted categories if their belief and ability do not change.

Let's first take a look at $L1$, $L4$ and $L2$ in SEQ-P2. The predictions of Lkb or Lka behavior are the same for these subjects. Their actual choices in SEQ-P2 and SEQ-P3, which are represented by the first bar (the dark one) in each category, share similar patterns with the simulated distributions. The actual distributions are less concentrated on the theoretically predicted categories, possibly as a result of higher deviation rates or less consistency across games. According to Table 5, exact tests of goodness-of-fit show that in three of these five cases ($L1$ in SEQ-P3, $L2$ in SEQ-P2 and $L4$ in SEQ-P3) the actual type distributions are not different statistically from the predicted ones at the significance level of 0.05 (Table 5). In the case of $L4$ subjects in SEQ-P2, the difference is at the 0.05 level but not the 0.01 level.

Since there might exist both $L2b$ and $L2a$ in SEQ-P3, and both $L3b$ and $L3a$ in SEQ-P2 and SEQ-P3, the type distributions of the $L2$ and $L3$ subjects could be different from the simulated ones. However, the difference should only be reflected in the predicted directions. Specifically, for $L2$'s behavior in SEQ-P3, the difference from the simulated distribution should only be in the proportions of best responses at player 2 position. Therefore small p-values should only occur in the "P4" and "P4P2" categories and the proportions of the other categories should not differ too much from the simulated ones. Similarly, for $L3$'s behavior in SEQ-P2 and SEQ-P3, the difference between actual data and simulated data should only be in the proportions of subjects best responding at player 1 position. As shown in columns (2)-(5) Table 5, the follow-up test of each category confirm that very few subjects fall outside of the predicted categories.

Thus it could be concluded that the simulated type distributions are good predictions of the aggregate pattern, which provides strong evidence that most subjects are consistent in belief and ability levels.

Result 2 About half of the $L2$- and $L3$-behaving subjects in the simultaneous games are best responding to the $L2$ or $L3$ belief, the rest are Lka types who could perform at most two or three steps of reasoning.

According to Figure 4, although there are a few $L2$ subjects playing the iterative domi-

	SEQ-P2	all categories	each category vs. the rest			
			P4	P4P3	P4P3P1	$L0$ + UI
$L1$	vs. predicted-$L1b$ & $L1a$	**0.0012**	**0.0040**	**0.0121**	0.0485	0.6249
$L2$	vs. predicted-$L2b$ & $L2a$	0.1702	0.7124	0.0704	0.5396	0.0648
$L3$	vs. predicted-$L3b$	**0.0000**	0.7669	**0.0000**	**0.0000**	0.2674
	vs. predicted-$L3a$	**0.0000**	0.5337	**0.0000**	**0.0000**	0.2674
$L4$	vs. predicted-$L4b$ & $L4a$	**0.0320**	0.7208	0.0329	**0.0093**	0.4522
	SEQ-P3	all categories	each category vs. the rest			
			P4	P4P2	P4P2P1	$L0$ + UI
$L1$	vs. predicted-$L1b$ & $L1a$	0.0824	0.1752	0.1111	0.4373	0.6249
$L2$	vs. predicted-$L2b$	**0.0000**	**0.0000**	**0.0000**	0.2676	0.3278
	vs. predicted-$L2a$	**0.0000**	**0.0000**	**0.0000**	0.0547	0.6334
$L3$	vs. predicted-$L3b$	**0.0000**	0.0791	**0.0006**	**0.0000**	0.2674
	vs. predicted-$L3a$	**0.0000**	0.5058	**0.0000**	**0.0000**	0.2674
$L4$	vs. predicted-$L4b$ & $L4a$	0.1842	0.1181	1.0000	0.0913	0.4453

Note: the numbers in bold fonts represents the test results at the 0.05 significance level. The first column shows the original tests including all four categories. Columns (2)-(5) show the follow-up tests of each category vs. the sum of all the other categories. Since there are four follow-up tests at the same time, the significance level is corrected by $0.05/4 = 0.0125$.

Table 5: p-values of the exact tests of goodness-of-fit

nant strategies as player 2 in SEQ-P3, which requires $L2$ belief and three steps of reasoning, more than half fail at this position. Especially, most of them best respond only as player 4, suggesting that they are bounded by two steps. The exact tests show that this group's behavior is significantly different from the simulated distribution assuming that they are all $L2b$ or $L2a$, with p-values less than 0.0001. The follow-up tests confirm that the deviations from the simulated distribution are confined to "P4" and "P4P2", but not to the rest of the categories, suggesting that such a pattern could not be due to deviations and misidentification.

A similar pattern could be found on the $L3$ subjects (Figure 5). The proportions of $L3b$, who best respond at all positions, and $L3a$, who could not best respond at player 1 position, appear to be half-half, and few subjects fall into other categories. The exact tests support that both distributions are significantly different from the simulated ones, mainly due to the differences in the "P4P3"/ "P4P2" and "P4P3P1"/ "P4P2P1" categories.

Overall, the evidence suggests that only half of the $L2$- and $L3$-behaving subjects in the simultaneous games are best responding to their belief. The rest of them use two or three steps of reasoning because this is the most they could do. Since there is evidence of both Lkb and Lka types, a model with only players' belief levels or their ability levels could not explain the behavior pattern.

Result 3 No difference has been found on $L3$'s behavior in SEQ-P2 and SEQ-P3, suggesting that the $L3b$ subjects hold belief in their opponents' belief not ability.

Now that there are traces of both Lkb and Lka types, it brings the question whether subjects' belief is in the opponents' belief or ability. That is, whether an Lkb subject believes that the opponents are bounded by $(k-1)$th-order belief or bounded by $k-1$ steps of reasoning.

It could be tested on the $L3$ subjects. In SEQ-P2, in order to play the iterative dominant strategy as player 1, $L3b$ needs to believe that the opponents think two steps, while in SEQ-P3 $L3b$ needs to believe that the opponents use three steps of thinking. If all $L3b$ subjects hold the belief that their opponents best respond to $L2$ belief, then SEQ-P3 and SEQ-P2 should make no difference for them, which serves as the null hypothesis in this test. If this is the case, there should be the same proportion of $L3$ subjects identified as $L3b$ in SEQ-P2 and SEQ-P3. Otherwise, if a subject believes that the opponents are $L2a$, he will not choose the iterative dominant strategies as player 1 in SEQ-P3. In this case, less $L3$ subjects would be identified as $L3b$ in SEQ-P3 than in SEQ-P2.

In the actual data, no difference could be found statistically in the type distributions from the two sets of sequential games (Figure 5). A closer look at the behavior patterns of the $L3$ subjects (Table 6) has confirmed that most of these subjects show the same $L3b$ or $L3a$ behavior in both SEQ-P3 and SEQ-P2. The pattern is consistent with the simulated distribution assuming that the subjects remain as the same $L3b$ or $L3a$ type in both sets of the sequential games. Therefore no evidence is found to reject the hypothesis that $L3b$'s belief is in the opponents' belief, not their reasoning steps.

It should be noted that with a sample size of 39 $L3$ subjects, to get a power higher than 0.8, it requires at least 25% of them responding to the belief that the opponents are bounded by two steps of thinking[6]. So there may very well exist a small number of such subjects, who could not be detected in this experiment due to lack of power.

[6] This result is calculated by G*Power. See Faul et al. (2007).

		behavior in SEQ-P2			
		P4	P4P3	P4P3P1	UI
in SEQ-P3	P4	3	2	1	0
	P4P2	0	9	4	1
	P4P2P1	0	5	10	1
	UI	0	1	1	1

Table 6: Behavioral patterns of the 39 $L3$ subjects in the two sequential games

4.4 Separation of Lkb and Lka Subjects

In this section each subject is assigned a type using the 20 first-mover choices from SIMUL and the two sequential games (excluding the 4 choices as second movers) with a likelihood function similar to (1). $L2b$ and $L3b$ could be separated from $L2a$ and $L3a$ using these three sets of games.

Six types ($L1$, $L2a$, $L2b$, $L3a$, $L3b$, $L4$) are included in the assignment process. There are four types who are always best responding to the Lk belief in all games. For the $L2b$ and $L3b$ types, by combining their behavior in the simultaneous and sequential games, they could be identified as having $b = 2$ and $b = 3$ respectively and strictly higher ability than the observed levels. For the $L1$ and $L4$ types, it could only be inferred $\min\{b, a\} \geq 1$ or $\min\{b, a\} \geq 4$ respectively, but it is not clear which factor is binding.

In addition, there are the $L2a$ and $L3a$ types, whose observed levels are determined by their ability. They behave like $L2b$ or $L3b$ in the simultaneous games, but could be separated from $L2b$ and $L3b$ by their behavior in the sequential games. $L2a$ subjects have $a = 2$ and $b \geq 2$, and choose the (iterative) dominant strategies only when it requires no more than two steps of reasoning. They differ from $L2b$ by not choosing the iterative dominant strategies at the player 2 positions of SEQ-P3. $L3a$ players have $a = 3$ and $b \geq 3$, and choose the (iterative) dominant strategies only when it requires no more than 3 steps of reasoning. They differ from $L3b$ by not best responding at the player 1 positions of SEQ-P2 and SEQ-P3.

Based on the evidence from the last subsection, I do not include a type who believes that the opponents are $L2a$, i.e. chooses the iterative dominant strategies at player 1 positions of SEQ-P2 but not SEQ-P3.

Subjects are assigned to a type with no more than 3 deviations from the type's predicted action profile. The results with 0-deviation or 6-deviation cutoffs are reported in the appendix. When a subject deviates too much from a predicted profile, he is assigned to the unidentifiable category if he chooses 5 out of the 6 dominating strategies at player 4 positions correctly, otherwise he is classified as $L0$. A randomly choosing subject has only 0.2% of a chance to correctly choose 5 dominant strategies.

		Baseline + Sequential Types								
		$L0$	$L1$	$L2a$	$L2b$	$L3a$	$L3b$	$\geq L4$	UI	sum
Baseline Types	L0	6	0	0	0	0	0	0	1	7
	L1	0	22	1	1	0	1	0	1	26
	L2	0	3	21	21	0	1	0	4	50
	L3	1	0	0	0	20	15	0	3	39
	L4	0	0	0	1	1	0	26	2	30
	UI	1	1	1	1	0	1	0	22	27
	sum	8	26	23	24	21	18	26	33	179

Note: $N = 179$. Subjects are assigned to a type with no more than 3 deviations. Otherwise they are assigned to $L0$ or unidentifiable. The subjects classified as unidentifiable are able to choose 5 out of 6 dominant strategies as player 4 but do not match any of the predicted patterns.

Table 7: Type assignment according to the observations from SIMUL, SEQ-P2 and SEQ-P3

Table 7 shows how the identified types change from using the 8 positions in the simultaneous games to using the 20 first-mover positions in the simultaneous plus sequential games. Most of the $L1$ and $L4$ subjects stay as the same level when more observations are included, and all of the $L0$ subjects in the simultaneous games are still identified as $L0$. Only approximately 15% of the subjects fall into a different category, but overall the majority of the subjects appear to be quite consistent across the three sets of games.

21 out of the 50 subjects who exhibit $L2$ behavior in the simultaneous games are identified as the $L2a$ type, who could do at most two steps of reasoning, compared with the other 21 $L2b$, who have higher ability and are always responding to $L2$ belief. Of the 39 $L3$ subjects in the simultaneous games, more than half are identified as $L3a$, and 15 are identified as the higher ability $L3b$ type. The result suggests that around half of the $L2$ and $L3$ behavior observed in the simultaneous games is due to lack of ability to think further, which validates the findings in Subsections 4.1 and 4.3.

4.5 CRT Scores and k-Levels

This subsection explores the correlation between subjects' identified types from the dominance-solvable games and the measures of their cognitive ability. The Cognitive Reflection Test is used as a quick measure of cognitive ability in this experiment. If the $L2b$ ($L3b$) subjects are more capable in reasoning than the $L2a$ ($L3a$) subjects as predicted in the theoretical model, the difference might also be reflected in their CRT scores.

The Cognitive Reflection Test is designed by Frederick (2005) test people's cognitive ability in decision making. It is composed of three short questions as follows:

(a) A bat and a ball cost 1.10 in total. The bat costs a dollar more than the ball. How much does the ball cost? ___ cents.

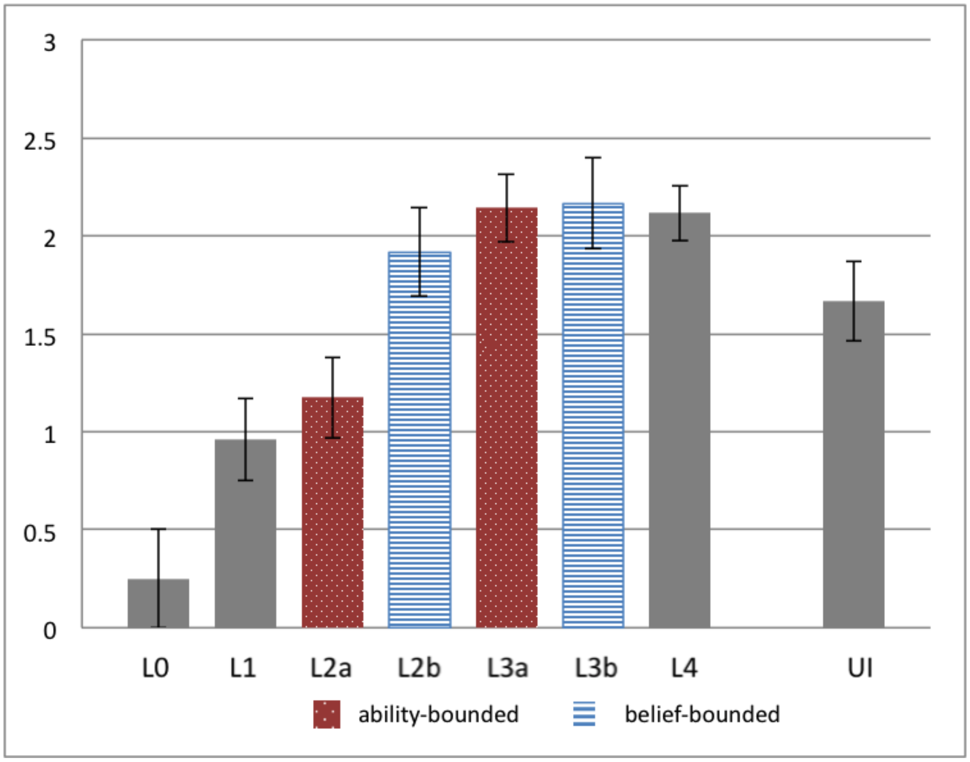
(b) If it takes 5 machines 5 min to make 5 widgets, how long would it take 100 machines to make 100 widgets? ___ min.

(c) In a lake, there is a patch of lily pads. Every day, the patch doubles in size. If it takes 48 days for the patch to cover the entire lake, how long would it take for the patch to cover half of the lake? ___ days.

All the three questions have intuitive but erroneous answers, and one needs to reflect on them for a brief moment to get them right.

Researches have found correlations of over 0.4 between CRT scores and other cognitive ability measure, such as the SAT, the Wonderlic Personnel Test, and the Vocabulary and Matrix Reasoning subtests (Frederick, 2005; Obrecht et al., 2009; Toplak et al., 2011). The CRT is also reported to relate to some important decision making characteristics. The subjects who score high in the CRT are more patience, and are more willing to take risk (Frederick, 2005). In addition, because the CRT tasks separate impulsive and reflective thinkers, it suggests that the ones who score higher in the test tend to do more rational thinking and are less likely to succumb to heuristics and bias. For example, Oechssler et al. (2009) find that higher CRT scores are correlated with lower conjunction fallacy and lower biases in updating probabilities. Toplak et al. (2011) discover that the CRT is a better predictor of the performance in a series of heuristics-and-biases tasks than other cognitive ability measures.

The participants in this experiment obtain a mean CRT score of 1.64, which is comparable to the results in the literature. In Figure 6 the mean CRT scores are displayed by types. Among the lower types ($L0$-$L2b$), subjects' CRT scores increase with their identified levels. Two-sided t-tests show that the differences are significant between these groups, except for $L1$ and $L2a$. The most striking result is that the $L2a$ subjects, who are identified to be of lower reasoning abilities, did much worse in CRT than the $L2b$ subjects, although both types exhibited the same behavior in the simultaneous games. The results of CRT and type identification corroborate each other, which implies that cognitive ability plays an important role in strategic reasoning.



Note: Standard errors are reported in the error bars.

Figure 6: Average CRT scores by types

The differences between high types ($L2b$-$L4$) are not that clear. T-tests show that the means are not different for these four groups. All of these subjects are able to do at least three steps of iterative thinking in the ring games, and their mean scores are pretty high. To serve as a comparison, Fredrick (2005) conducted the CRT on 3,428 subjects, primarily college students, and found a mean score of 1.24. The mean scores from top universities like Princeton and Harvard are around 1.4-1.6. Only MIT students get a high mean of over 2. Therefore, one possible explanation of why higher types are not separable by CRT scores could be that the CRT results do not distinguish between the subjects whose reasoning abilities have reached a certain high level.

Another interesting observation is the difference between $L0$ and unidentifiable subjects. The mean CRT score of $L0$ subjects is close to 0. Actually, 7 out of 8 subjects in this group scored 0 in the test. Contrastingly, the unidentifiable subjects perform much better and their average score is close to the mean of the whole sample. It supports the separation of these unidentifiable subjects from the irrational $L0$. The unidentifiable subjects could be of high cognitive ability, but do not follow the prediction of the level-k model. Or it could be that they are inconsistent and act as different types across different games.

I further run multilogit regressions to determine whether subjects' levels could be ex-

Independent Variable: CRT scores							
base outcome	$L0$	$L1$	$L2a$	$L2b$	$L3a$	$L3b$	$L4$
vs $L0$		1.440	1.570	2.471**	2.717***	2.814***	2.867***
		(1.028)	(0.966)	(1.042)	(1.024)	(1.043)	(1.019)
vs $L1$	-1.440		0.130	1.031***	1.277***	1.373***	1.426***
	(1.028)		(0.349)	(0.385)	(0.339)	(0.362)	(0.333)
vs $L2a$	-1.570	-0.130		0.901**	1.147***	1.243***	1.297***
	(0.966)	(0.349)		(0.353)	(0.340)	(0.365)	(0.310)
vs $L2b$	-2.471**	-1.031***	-0.901**		0.246	0.342	0.396
	(1.042)	(0.385)	(0.353)		(0.325)	(0.360)	(0.319)
vs $L3a$	-2.717***	-1.277***	-1.147***	-0.246		0.0962	0.150
	(1.024)	(0.339)	(0.340)	(0.325)		(0.324)	(0.286)
vs $L3b$	-2.814***	-1.373***	-1.243***	-0.342	-0.0962		0.0534
	(1.043)	(0.362)	(0.365)	(0.360)	(0.324)		(0.328)
vs $L4$	-2.867***	-1.426***	-1.297***	-0.396	-0.150	-0.0534	
	(1.019)	(0.333)	(0.310)	(0.319)	(0.286)	(0.328)	

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Standard errors are reported in the parentheses. Each row shows a regression with a different type being the base outcome. Session fixed effects are included in the multilogit regressions. Unidentifiable subjects are excluded.

Table 8: Multilogit regression of types on CRT scores

plained by their performances in CRT[7]. Table 8 reports 7 regressions of subjects' identified types on CRT scores, with different types as base outcomes. For example, the first row shows the coefficients of CRT on the probability of being each type versus being $L0$. The regressions show that the lower types ($L0$, $L1$ and $L2a$) are distinguishable from the higher types ($L2b$, $L3a$, $L3b$ and $L4$) using their CRT scores. However, the differences within these two groups are not significant.

[7]Similar analysis was done by Georganas et al. (2015). They do not find a monotone relationship between levels and CRT scores as seen in this study. But their results point to a link between CRT scores and earnings, which could also be observed in my data.

4.6 Robustness Check: Learning Effect

An essential assumption in the above analysis is that each subject's belief level and ability level remain constant throughout the experiment, with a few deviations due to random preference shift or trembling hand. However, if certain learning effects prevailed, the subjects became more proficient in locating iterative dominant strategies as the experiment proceeded. Then the observed pattern could be driven by the order of games in the experiment. For example, the subjects who played the more difficult positions, such as player 1 or player 2 positions, in the later periods would have a larger chance of figuring them out. If this is the case, the higher types would not be the group with higher ability, but the group with more opportunity to learn.

A detailed analysis of learning effects will be reported in the appendix. In summary, I find mild learning effects on a couple of player positions, but the type classification results do not appear to be affected by learning. This subsection compares the type distributions of the whole sample and the groups who might have an advantage in learning, showing that learning has almost no effect in shaping the type distribution.

Given the special structure of the ring games, it might be advantageous to play the player 4 positions first, for it could help the subjects to figure out early in the experiment that the games could be solved by iterative dominance. In addition, playing as second movers might also help, for it motivates the subject to look into the dependency relationships between him and the opponents. If this is true, then the subjects who played more player 4 positions or second movers in the earlier stage would be more likely to behave like higher types. To address this concern, I check whether the performances of these subjects differ from the whole sample.

In each session, subjects played the 24 games in different random orders. Table 9 gives the number of subjects who played more player 4 positions and second mover positions in the earlier 12 games. The numbers of player 4 positions are denoted by $n(P4)$, and the number of second mover positions by $n(PSM)$. There are 44 subjects who played at more than four player 4 positions in the earlier 12 games, and 53 subjects who played at more than six player 4 or second mover positions. These subjects are regarded as having advantages in learning. The following results still hold if other cutoffs are used to determine advantages.

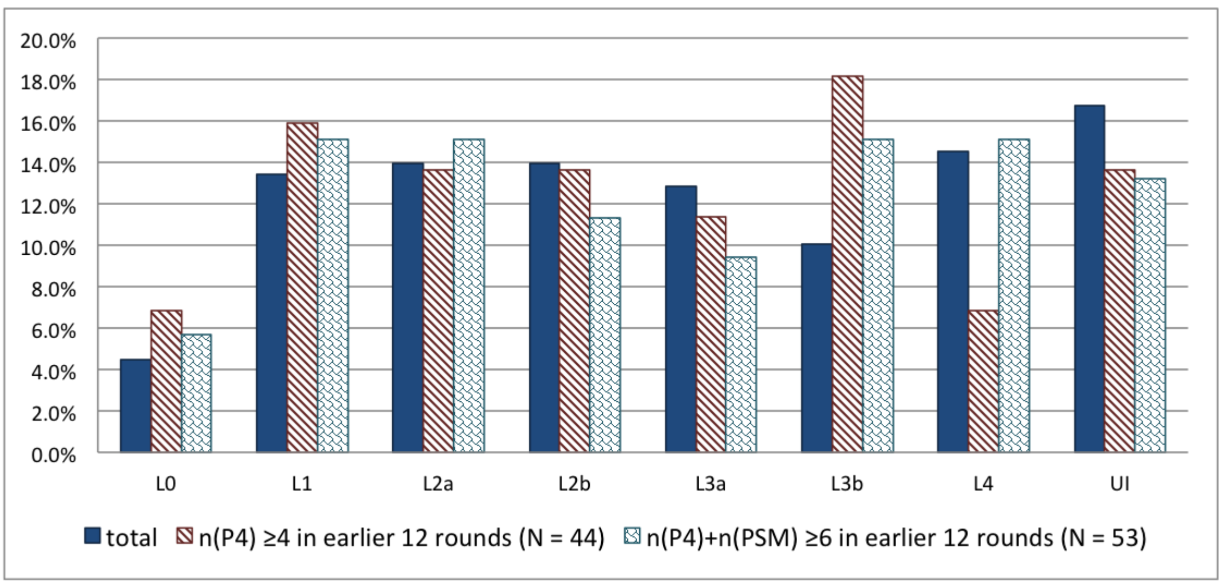
As shown in Figure 7, no evidence has been found that they performed better than the rest of the subjects. The distributions of these groups are not shifted toward the higher types. The goodness-of-fit tests show that the type distribution of the subjects with $n(P4) + n(PSM) \geq 6$ is not statistically different from the distribution of the whole sample, and that for the group with $n(P4) \geq 4$ the differences only occur in the $L3b$ and

$n(P4)$	≥ 1	≥ 2	≥ 3	≥ 4	≥ 5	≥ 6
N	179	155	124	44	31	0
$n(P4) + n(PSM)$	≥ 3	≥ 4	≥ 5	≥ 6	≥ 7	≥ 8
N	179	154	108	53	36	7

Note: $n(P4)$ denotes the number of player 4 positions played by the subject in the earlier 12 games. $n(PSM)$ denotes the number of second mover positions played by the subject in the earlier 12 games. The highlighted cells represent the cutoffs used in the following analysis.

Table 9: Number of subjects who played more advantageous positions in earlier 12 games

$L4$ categories. Therefore it is safe to say that the identified patterns of the main results are not affected by subjects' learning of iterative dominance.



Note: $n(P4)$ denotes the number of player 4 positions played by the subject in the earlier 12 games. $n(PSM)$ denotes the number of second mover positions played by the subject in the earlier 12 games.

Figure 7: Type distributions of the subjects who have an advantage in learning

5 Conclusion

This paper reports an experiment to separate the high ability subjects (Lkb) who behave as Lk due to their beliefs and the low ability subjects (Lka) who could think at most k steps. The separation happens at certain first mover positions of the sequential ring games, where it requires three or four steps to respond to $L2$ or $L3$ belief. The $L2b$ or $L3b$ subjects are still able to best respond to the same $L2$ or $L3$ belief as they did in the simultaneous games. But the $L2a$ or $L3a$ ones could do at most two or three steps of reasoning, and thus could not handle the one more step.

I find that most subjects behave consistently, in the sense of belief levels and ability levels, across the three sets of ring games. Their behavioral patterns in the sequential games fit theoretical predictions. Out of the 50 and 39 subjects classified as $L2$ and $L3$ from their choices in the simultaneous games, around half have reached their upper boundaries of reasoning. In addition, evidence on $L3$ subjects supports that their beliefs are on their opponents' belief levels but not reasoning steps. Finally, the CRT scores are significantly higher for the high ability $L2b$ than the low ability $L2a$, which supports the separation of the two types. But higher types ($L2$, $L3a$, $L3$ and $L4$) are not distinguishable using CRT scores.

The findings suggest large heterogeneity in subjects' abilities to best respond to even low order belief. The observed low levels in the previous studies could be explained by both the presence of low ability types and the low-order beliefs of high ability types. Although the high types have incorrect beliefs, their low-order beliefs are not entirely unfounded, given the large proportion of the cognitively bounded subjects.

The existing literature has demonstrated the descriptive power of the level-k model. To make it also an explanatory and predictive model, it requires a better understanding of why people behave as certain levels, or where people get their belief from. For example, the existence of ability-bounded subjects in this study shows that a lot of people might not start with a clear idea of the opponents' levels. Rather, their belief could be formed through the anchoring and adjusting process suggested by Brandenburger and Li (2015), and this process would stop when they have reached their cognitive boundaries.

References

[1] Agranov, Marina, Elizabeth Potamites, Andrew Schotter and Chloe Terigiman. 2012. "Beliefs and Endogenous Cognitive Levels: An Experimental Study." *Games and Economic Behavior* 75: 449-63.

[2] Alaoui, Larbi and Antonio Panta. 2015. "Endogenous Depth of Reasoning." *The Review of Economic Studies* advance access published December 4, 2015, doi:10.1093/restud/rdv052

[3] Arad, Ayala and Ariel Rubinstein. 2012. "The 11-20 Money Request Game: A Level-k Reasoning Study". *American Economic Review* 102(7): 3561-73.

[4] Bhatt, Meghana and Colin F. Camerer. 2005. "Self-referential Thinking and Equilibrium as States of Mind in Games: fMRI Evidence." *Games and Economic Behavior* 52: 424-59.

[5] Branas-Garza, Pablo, Teresa Garcia-Munoz and Roberto Hernan Gonzalez. 2012. "Cognitive Effort in the Beauty Contest Game." *Journal of Economic Behavior and Organization* 83: 254-260.

[6] Brandenburger, Adam and Xiaomin Li. 2015. "Thinking about Thinking and Its Cognitive Limits." Working paper.

[7] Breitmoser, Yves, Jonathan H.W. Tan and Daniel John Zizzo. 2014. "On the Beliefs off the Path: Equilibrium Refinement due to Quantal Response and Level-k." *Games and Economic Behavior* 86: 102-25.

[8] Brocas, Isabelle, Juan D. Carriloo, Stephanie W. Wang and Colin F. Camerer. 2014. "Imperfect Choice or Imperfect Attention? Understanding Strategic Thinking in Private Information Games." *Review of Economic Studies* 81: 944-70.

[9] Burchardi, Konrad B. and Stefan P. Penczynski. 2014. "Out of Your Mind: Eliciting Individual reasoning in One Shot Games." *Games and Economic Behavior* 84: 39-57.

[10] Camerer, Colin F., Teck-Hua Ho, and Juin Kuan Chong. 2004. "A Cognitive Hierarchy Model of Games." *Quarterly Journal of Economics*, 119(3): 861-898.

[11] Costa-Gomes, Miguel A., Vincent P. Crawford and Bruno Broseta. 2001. "Cognition and Behavior in Normal-Form Games: An Experimental Study." *Econometrica* 69(5): 1193-1235.

[12] Costa-Gomes, Miguel A., and Vincent P. Crawford. 2006. "Cognition and Behavior in Two-Person Guessing Games: An Experimental Study." *American Economic Review* 96(5): 1737-68.

[13] Costa-Gomes, Miguel A., and Georg Weizsacker. 2008. “Stated Beliefs and Play in Normal-Form Games” *Review of Economic Studies* 75: 729-62.

[14] Crawford, Vincent P., Miguel A. Costa-Gomes, and Nagore Iriberri. 2013. “Structural Models of Nonequilibrium Strategic Thinking: Theory, Evidence, and Applications.” *Journal of Economic Literature* 51(1):5-62.

[15] Faul, Franz, Edgar Erdfelder, Albert-Georg Lang, and Axel Buchner. 2007. “G*Power 3: A flexible statistical power analysis program for the social, behavioral, and biomedical sciences.” *Behavior Research Methods* 39: 175-191.

[16] Friedenberg, Amanda, Willemien Kets, and Terri Kneeland. 2016. “Cognition and Rationality.” Working paper.

[17] Frederick, Shane. 2005. “Cognitive Reflection and Decision Making.” *Journal of Economic Perspectives* 19: 25-42.

[18] Gchter, Simon and Elke Renner. 2010. “The Effects of (Incentivized) Belief Elicitation in Public Goods Experiments.” *Experimental Economics* 13: 364-377

[19] Georganas, Sotiris, Paul J. Healy and Roberto A. Weber. 2015. “On the Persistence of Strategic Sophistication.” *Journal of Economic Theory* 159: 369?400..

[20] Gill, David and Victoria Prowse. 2015. “Cognitive Ability, Character Skills, and Learning to Play Equilibrium: A Level-k Analysis.” *Journal of Political Economy*: forthcoming.

[21] Ho, Teck-Hua, Colin F. Camerer, and Keith Weigelt. 1998. “Iterated Dominance and Iterated Best Response in Experimental '*p*-Beauty Contests'.” *American Economic Review*, 88(4): 947-69.

[22] Ho, Teck-Hua, and Xuanming Su. 2013. "A Dynamic Level-k Model in Sequential Games.” *Management Science* 59(2):452-69.

[23] Jin, Ye. 2016. “A Comment on ‘Identifying Higher-order Rationality”. Working paper.

[24] Kneeland, Terri. 2015.“Identifying Higher-order Rationality”. *Econometrica* 83: 2065-2079.

[25] Manski, Charles F. and Claudia Neri. 2013. “First- and Second-Order Subjective Expectations in Strategic Decision-Making: Experimental Evidence.” *Games and Economic Behavior*. 81: 232-54.

[26] Nagel, Rosemarie. 1995. "Unraveling in Guessing Games: An Experiment Study.” *American Economic Review*, 85(5): 1313-1326.

[27] Nyarko, Yaw and Andrew Schotter. 2002. "An Experimental Study of Belief Learning Using Elicited Beliefs." *Econometrica* 70: 971-1005.

[28] Obrecht, Natalie A., Gretchen B. Chapman and Rochel Gelman. 2009. "An encounter frequency account of how experience affects likelihood estimation." *Memory & Cognition* 37: 632-643.

[29] Oechssler, Jorg, Andreas Roider and Patrick W. Schmitz. 2009. "Cognitive abilities and behavioral biases." *Journal of Economic Behavior and Organization* 72: 147-152.

[30] Palacios-Huerta, Ignacio, and Oscar Volij. 2009. "Field Centipedes." *American Economic Review*, 99(4): 1619-35.

[31] Rutstrom, E. Elisabet and Nathaniel T. Wilcox. 2009. "Stated Beliefs versus Inferred Beliefs: A Methodological Inquiry and Experimental Test." *Games and Economic Behavior* 67: 616-32.

[32] Schnusenberg, Oliver and Andres Gallo. 2011. "On cognitive ability and learning in a beauty contest." *Journal for Economic Educators* 11: 13-24.

[33] Slonim, Robert L. 2005. "Competing Against Experienced and Inexperienced Players." *Experimental Economics* 8:55-75.

[34] Stahl, Dale O., and Ernan Haruvy. 2008. Level-n Bounded Rationality in Two-player Two-stage Games." *Journal of Economic Behavior and Organization* 65: 41-61.

[35] Stahl, Dale O., and Paul W. Wilson. 1994. "Experimental Evidence on Players' Models of Other Players." *Journal of Economic Behavior and Organization* 25(3): 309-27.

[36] Stahl, Dale O., and Paul W. Wilson. 1995. "On Players' Models of Other Players: Theory and Experimental Evidence." *Games and Economic Behavior* 10(1): 218-54.

[37] Strzalecki, Tomasz. 2014. "Depth of Reasoning and Higher Order Beliefs". *Journal of Economic Behavior and Organization* 108: 108-22.

[38] Toplak, Maggie. E., Richard F. West, and Keith E. Stanovich. 2011. "The Cognitive Reflection Test as a predictor of performance on heuristics-and-biases tasks." *Memory & Cognition* 39: 1275-1289.

APPENDIX

A Type Assignment with the Assumption of Uniform Randomizing $L0$

The assumption used to predict Lk behavior in the ring games is that an Lk player does not respond to the changes in $(k+1)$- or higher-order payoffs (**ER** in Kneeland (2015)). An alternative assumption, which is widely used in Lk experiments, is that the irrational $L0$ type uniformly randomize on all the possible actions (**UP**: uniform prior on $L0$).

		$L0$	$L1$	$L2$	$L3$	$L4$	UI
SIMUL	ER	7	26	50	39	30	27
		3.9%	14.5%	27.9%	21.8%	16.8%	15.1%
	UP	6	10	43	39	53	28
		3.4%	5.6%	24.0%	21.8%	29.6%	15.6%
		$L0$	P4	P4P3	P4P3P1	-	UI
SEQ-P2	ER	7	43	69	46	-	14
		3.9%	24.0%	38.5%	25.7%	-	7.8%
	UP	7	32	60	78	-	2
		3.9%	17.9%	33.5%	43.6%	-	1.1%
		$L0$	P4	P4P2	P4P2P1	-	UI
SEQ-P3	ER	5	68	46	45	-	15
		2.8%	38.0%	25.7%	25.1%	-	8.4%
	UP	5	22	46	94	-	12
		2.8%	12.3%	25.7%	52.5%	-	6.7%

Note: $N = 179$. Subjects are assigned to a type with no more than 1 deviation when using ER, with no more than 2 deviations when using UP. Otherwise they are assigned to $L0$ or unidentifiable. The subjects classified as unidentifiable are able dominant strategies as Player 4 but do not match any of the predicted patterns.

Table 10: Type assignment using the two assumptions ER and UP

The distribution of assigned types using the predicted Lk behavior of UP is given in Table 10. The 2-deviation cutoff is used here, so a random choosing subject only has a probability of less than 5% of being assigned to one of the Lk types, which is comparable to the main results. The distributions shift to the right in all three sets of games. In the simultaneous games, the number of $L4$ subjects almost doubles, and the number of $L1$ drops by more than a half. The shift is even larger in SEQ-P3. The number of subjects assigned to "P4" category under UP is less than a third of the number under ER. And the number in "P4P2P1" category increases by more than 100%. A closer look at the type changes confirms that everyone's level rises or at least stays the same. Nobody goes to a lower level under UP. Nevertheless, the numbers of $L0$ and the unidentifiable are quite close. There are 6 subjects classified as $L0$ and 14 classified as the unidentifiables under both assumptions.

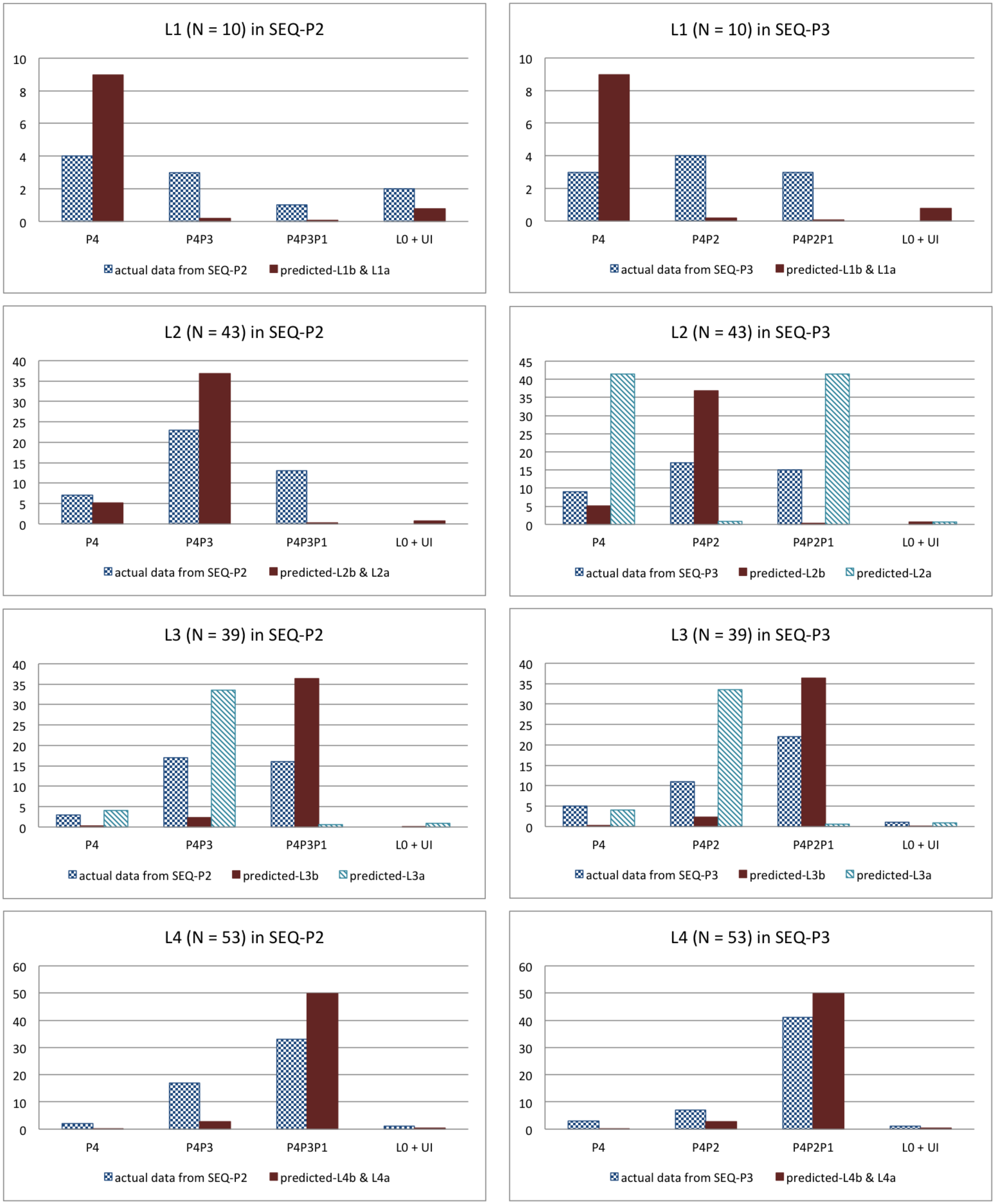
I try to perform the same analysis of each type's behavioral pattern in the sequential games (Figure 8). However, the overestimation of lower types in the two sequential games makes the treatment effects less clear. A large number of $L1$- and $L2$-behaving subjects in SIMUL are classified as "P4P2P1" in the sequential games, which is inconsistent with the theoretical prediction. It is impossible to tell how much subjects are Lka with such large inconsistency across the three sets of games. The overestimation is less severe among higher types, and a similar pattern could be observed on $L3$ and $L4$ subjects as in the main analysis.

Type classification with UP tends to overestimate the lower types. This is because the predicted action profile of each type on the off-equilibrium path using UP is a special case of that using ER. Since UP puts stronger restrictions on the off-equilibrium path, it is more difficult to match a subject to a low type. The overestimation would be more severe if fewer subjects follow the prediction of UP, as observed in SEQ-P3. Hence I use ER in the main analysis, which I believe provides more robust and reliable results.

B Type Classification Results Allowing for 0 or 2 Deviations

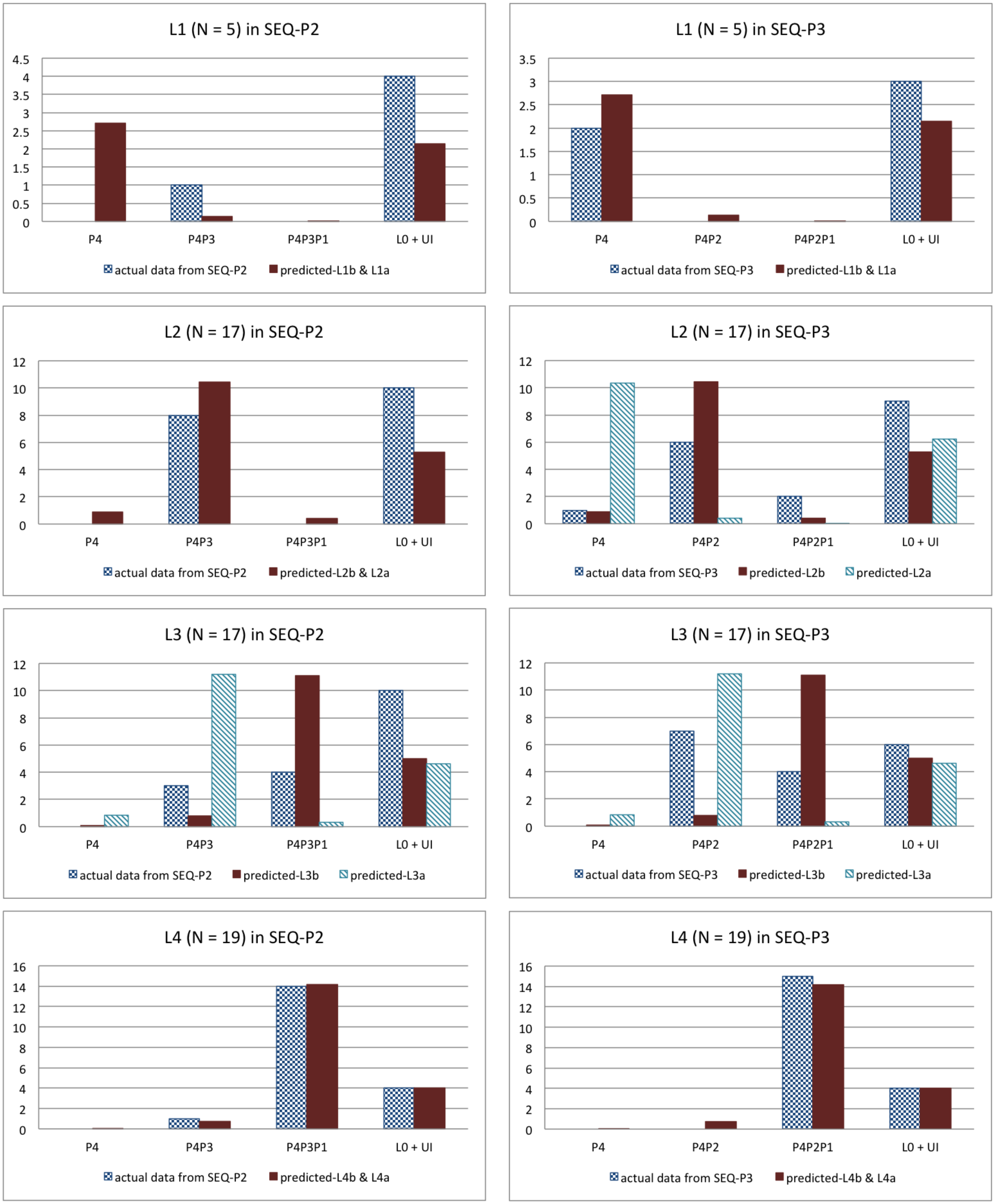
Table 4 has shown what the type distribution looks like allowing for 0 or 2 deviations. If no deviation is allowed for during the type assignment process, there will be a large fraction of subjects (113 out of 184) who could not be put into any of the $L0$-$L4$ categories.With the 0-deviation cutoff, since few subjects' behavior profile could be matched to a certain type with 0 deviation in all three sets of games, it is hard to determine a clear pattern of each type's behavior in the sequential games. As shown in Figure 9 , although most $L4$ subjects behave as predicted in both sequential games, half of the $L1$, $L2$ and $L3$ subjects fall into the unidentifiable category. So it is less clear what proportion of the subjects are best responding to their belief and what proportion are bounded by reasoning ability in the sequential games.

In the type classification using all three sets of games, most subjects remain unidentified with the 0-deviation cutoff (Table 11).



Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 ("P4"), at P4 and P3 ("P4P3"), and at P4, P3 and P1 ("P4P3P1"). In SEQ-P3, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 ("P4"), at P4 and P2 ("P4P2"), and at P4, P2 and P1 ("P4P2P1"). Each subject is assigned to a category with UP and 2-deviation cutoff. The dark blue bars give the actual type distribution. The light blue and medium blue bars give the simulated type distribution assuming that these subjects are *Lkb* or *Lka*.

Figure 8: Behavioral patterns with the uniform prior assumption (UP)



Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 ("P4"), at P4 and P3 ("P4P3"), and at P4, P3 and P1 ("P4P3P1"). In SEQ-P3, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 ("P4"), at P4 and P2 ("P4P2"), and at P4, P2 and P1 ("P4P2P1"). Each subject is assigned to a category with ER and 0-deviation cutoff. The dark blue bars give the actual type distribution. The light blue and medium blue bars give the simulated type distribution assuming that these subjects are Lkb or Lka.

Figure 9: Behavioral patterns in sequential games (0 deviation)

		Baseline + Sequential Types								
		$L0$	$L1$	$L2a$	$L2b$	$L3a$	$L3b$	$\geq L4$	UI	sum
Baseline Types	L0	7	0	0	0	0	0	0	1	8
	L1	0	0	0	0	0	0	0	5	5
	L2	0	0	1	3	0	0	0	14	18
	L3	0	0	0	0	2	2	0	13	17
	L4	0	0	0	0	0	0	12	7	19
	UI	1	0	0	0	0	0	0	111	112
	sum	8	0	1	3	2	2	12	151	179

Note: $N = 179$. Subjects are assigned to a type with 0 deviation.

Table 11: Type assignment according to the observations from SIMUL, SEQ-P3 and SEQ-P2 (0 deviation)

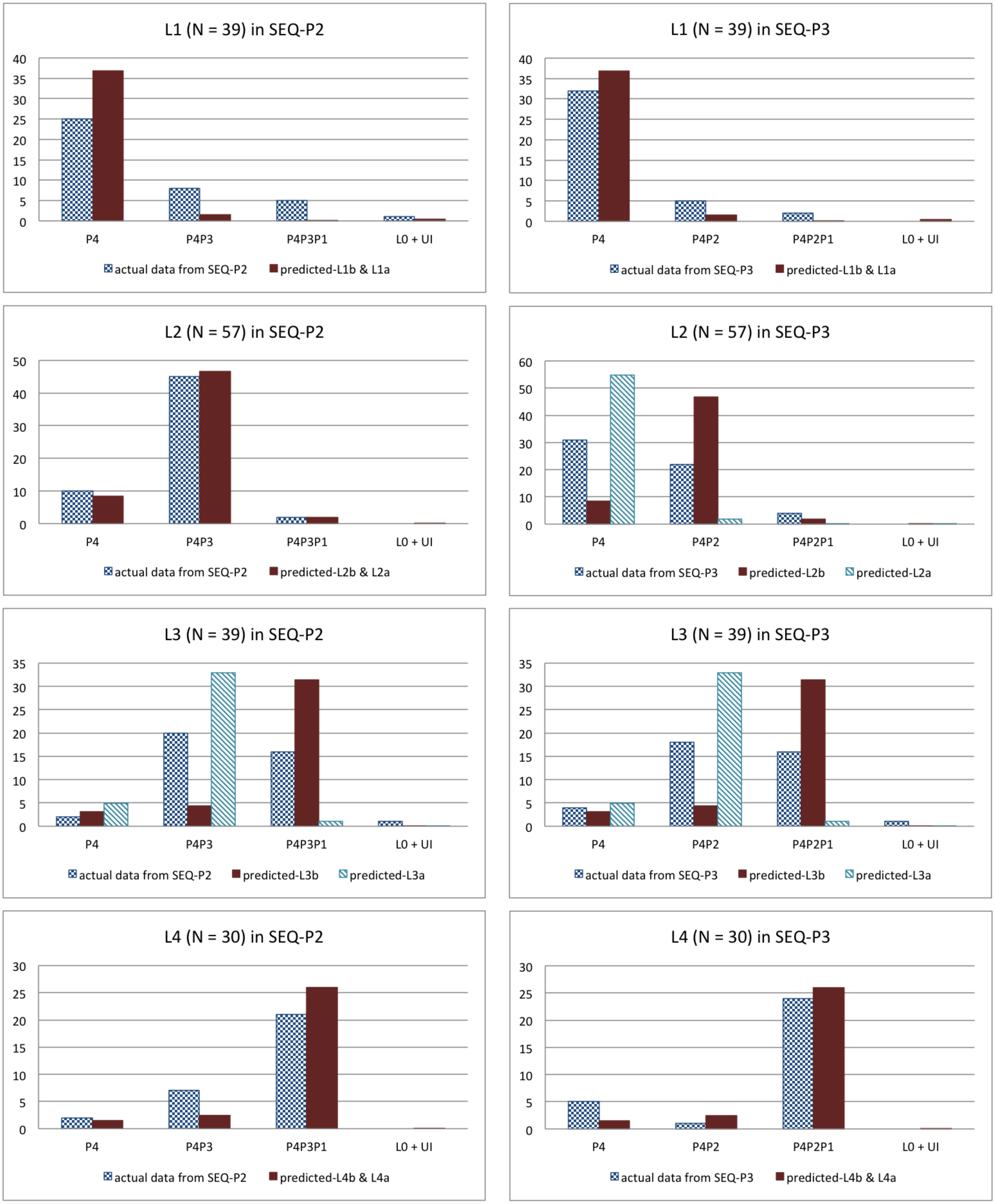
When allowing for 2 instead of 1 deviation in the matching process, the risk of misidentifying the randomly choosing subjects to an Lk type increases. More specifically, according to Table 4, 15% of the $L0$ players who used to be classified as $L0$ and 5% who used to be classified as unidentifiable are now assigned to one of the $L1$-$L4$ categories. When turning to the actual data, a decrease in the number of unidentifiables could be observed. But it is hard to tell how many of the subjects leaving the unidentifiable group are the misidentified $L0$ or the real Lk types with larger deviation rates. The change in the $L0$ category might provide some clues. There are only 2 subjects leaving the $L0$ category when switched to the 2-deviation cutoff. So the increase in the misidentification of $L0$ appears not to be a big problem in my data. Therefore it is helpful to take a look at the results with the 2-deviation cutoff, in which almost all the subjects could be classified.

Most previously unidentified subjects in the simultaneous games are moved to one of the lower types ($L1$ and $L2$) when allowing for 2 deviations, and there is no change in the number of $L3$ and $L4$ subjects. The behavioral pattern of each type in sequential games appears to be similar as in the main analysis (Figure 10). But it should be noted that since it identifies more lower types from the previously unidentifiable pool by using the 2-deviation cutoff, the use of this cutoff picks up slightly more of the ability-bounded $L2a$ and $L3a$ types versus the belief-bounded $L2b$ and $L3b$ types in Table 12.

		Baseline + Sequential Types								
		$L0$	$L1$	$L2a$	$L2b$	$L3a$	$L3b$	$\geq L4$	UI	sum
Baseline Types	L0	0	5	0	0	0	0	0	0	5
	L1	1	30	4	2	0	2	0	0	39
	L2	0	2	29	24	0	2	0	0	57
	L3	1	0	0	0	23	15	0	0	39
	L4	0	1	1	1	1	0	26	0	30
	UI	1	7	0	1	0	0	0	0	9
	sum	3	45	34	28	24	19	26	0	179

Note: $N = 179$. Subjects are assigned to a type with no more than 6 deviations. Otherwise they are assigned to $L0$ or unidentifiable. The subjects classified as unidentifiable are able to choose 5 out of 6 dominant strategies as player 4 but do not match any of the predicted patterns.

Table 12: Type assignment according to the observations from SIMUL, SEQ-P3 and SEQ-P2 (6 deviations)



Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 ("P4"), at P4 and P3 ("P4P3"), and at P4, P3 and P1 ("P4P3P1"). In SEQ-P3, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 ("P4"), at P4 and P2 ("P4P2"), and at P4, P2 and P1 ("P4P2P1"). Each subject is assigned to a category with ER and 2-deviation cutoff. The dark blue bars give the actual type distribution. The light blue and medium blue bars give the simulated type distribution assuming that these subjects are *Lkb* or *Lka*.

Figure 10: Behavioral patterns in sequential games (2 deviations)

C Additional Analysis on Learning Effects

A thorough analysis on learning effects is reported in this section. I start by checking whether there exist systematic type shifts in the data. That is, whether subjects are more likely to shift to a higher (lower) type in the later (earlier) period of the experiment. Since the choices in all 20 first mover positions are needed to pin down a subject's type, it is not possible to estimate one's type in the early and late parts of the experiment separately. Instead, I look at the deviations from the choice pattern of one's assigned type. The deviation observed in the type assignment process could be sorted into one of three cases: (1) a non-equilibrium strategy is chosen at a position where the type should have chosen an equilibrium strategy. (2) two different strategies are chosen at the same position of the two paired rings where the type should be on off-equilibrium path and choose the same strategy, and at least one of the two strategies chosen is an equilibrium strategy. (3) two different strategies are chosen at the same position of the two paired rings where the type should be on off-equilibrium path and choose the same strategy, and neither of the two strategies chosen is an equilibrium strategy.

Case (1) implies that the subject might shift to a lower level when playing that game and Case (2) corresponds to the shift to a higher level. If learning affects subjects' behavior, they should be more likely to deviate to a higher level in the later half of the experiment and more likely to a lower level in the earlier half. If, however, the growth of fatigue plays a more important role, it should be opposite. Of course it could not be ruled out that some deviations in Case (1) and (2) are caused by preference shifts or mistakes. But if the occurrences of preference shifts or mistakes are assumed to be time-invariant, then they could be canceled out when only the differences of the earlier and later halves are examined.

	8 Baseline Games			20 First Mover Games		
	Case (1)	Case (2)	Case (3)	Case (1)	Case (2)	Case (3)
Shifts	downward	upward	-	downward	upward	-
Earlier	12	26	25	42	80	53
Later	2	27		24	79	

Note: the first two cases are counted in the earlier and later 12 games respectively. $L0$ and unidentifiable subjects are excluded.

Table 13: Deviations sorted into three cases

Table 13 reports the deviations of the classified subjects, from both the 8 SIMUL games and the 20 first mover games type assignments. $L0$ and unidentifiable subjects are excluded from this analysis, because according to the definition $L0$s could be using any combination of strategies, and since I could not identify the decision rules of the unidentifiable it would be hard to determine which choices are deviations from their rules. The 24 games were played in a random order and the orders of play were different for each subject. Cases (1) and (2) could therefore be put into two categories, that is whether these deviations occur in the first or later 12 games. In Case (3) it could only be observed that subjects are choosing two different strategies at the same position, but it is impossible to tell which one is the deviation (or both could be deviations). So only the total numbers of deviations in Case (3) are reported.

In Case (1), players deviate to a lower type. This kind of deviation is more likely to happen in the first half of the experiment, suggesting some sort of learning effects. However, the occurrences of Case (2) deviations, which imply a shift to a higher level, are quite close between earlier and later periods of the experiment.

I next run a probit regression to determine the learning effect specifically at each position.

$$\text{Probit}(Y_i) = \alpha + \beta_1 \text{L12}_i + \beta_2 \text{POS}_i + \beta_{3j} \text{POS}_{ij} \times \text{L12}_i + \epsilon_i, \tag{2}$$

where $Y_i = 1$ when an equilibrium strategy is chosen, and $Y_i = 0$ otherwise; $L12_i = 1$ if that choice is made in the later 12 games, and $L12_i = 0$ otherwise; POS_{ij} denotes the position dummy at position j. The session fixed effects are also controlled.

Table 14 reports the coefficients $\beta_1 + \beta_{3j}$ of each position j. One position dummy, player 4 of G5, is dropped because of collinearity. Significant positive learning effects are found at only 3 of the 23 positions.

Since the identification uses at least a pair of ring games, the learning effects at three positions is unlikely to affect the type distribution. It is reported in Section 4.6 that the performance of the subjects who could have better opportunities to learn is not different from the whole sample. Here I further show with multilogit regressions that playing more player 4 positions or second mover positions in the earlier periods does not affect the probability of being assigned to a high type (Table 15 and Table 16).

		player 1	player 2	player 3	player 4
Baseline	G1	0.215	0.009	0.287	0.127
		(0.193)	(0.189)	(0.210)	(0.389)
	G2	0.267	0.055	0.235	0.010
		(0.190)	(0.188)	(0.211)	(0.417)
		player 1	player 2 (2nd)	player 3	player 4
Seq-P2	G3	0.210	0.108	0.570**	-0.226
		(0.193)	(0.331)	(0.234)	(0.365)
	G4	0.232	-0.683	-0.103	0.272
		(0.199)	(0.427)	(0.227)	(0.315)
		player 1	player 2	player 3 (2nd)	player 4
Seq-P3	G5	0.028	0.492**	0.142	-
		(0.187)	(0.200)	(0.344)	-
	G6	0.393**	-0.144	0.237	0.010
		(0.193)	(0.197)	(0.369)	(0.395)

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Standard errors are reported in the parentheses. (2nd) denotes second stage movers. Player 4 of G5 is dropped due to collinearity.

Table 14: Experience effect at each position ($\beta_1 + \beta_{3j}$)

Independent Variable: $I(n(P4) > 4$ in earlier 12 games)							
base outcome	*L0*	*L1*	*L2a*	*L2b*	*L3a*	*L3b*	*L4*
vs *L0*		-0.398	-0.713	-0.770	-0.843	0.182	-1.713*
		(0.883)	(0.897)	(0.887)	(0.893)	(0.884)	(0.999)
vs *L1*	0.398		-0.315	-0.372	-0.445	0.581	-1.315
	(0.883)		(0.720)	(0.670)	(0.715)	(0.656)	(0.823)
vs *L2a*	0.713	0.315		-0.0564	-0.129	0.896	-1.000
	(0.897)	(0.720)		(0.682)	(0.703)	(0.713)	(0.832)
vs *L2b*	0.770	0.372	0.0564		-0.0730	0.952	-0.943
	(0.887)	(0.670)	(0.682)		(0.697)	(0.676)	(0.811)
vs *L3a*	0.843	0.445	0.129	0.0730		1.025	-0.870
	(0.893)	(0.715)	(0.703)	(0.697)		(0.723)	(0.835)
vs *L3b*	-0.182	-0.581	-0.896	-0.952	-1.025		-1.895**
	(0.884)	(0.656)	(0.713)	(0.676)	(0.723)		(0.801)
vs *L4*	1.713*	1.315	1.000	0.943	0.870	1.895**	
	(0.999)	(0.823)	(0.832)	(0.811)	(0.835)	(0.801)	

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Standard errors are reported in the parentheses. Each row shows a regression with a different type being the base outcome. Session fixed effects are included in the multilogit regressions. Unidentifiable subjects are excluded.

Table 15: Multilogit regression of types on Learning Effects (1)

Independent Variable: $I(n(P4) + N(PSM) > 6$ in earlier 12 games)							
base outcome	$L0$	$L1$	$L2a$	$L2b$	$L3a$	$Lb3$	$L4$
vs $L0$		0.381	-0.320	-0.633	-0.740	0.689	-0.761
		(0.893)	(0.907)	(0.926)	(0.948)	(0.951)	(0.922)
vs $L1$	-0.381		-0.700	-1.014	-1.121	0.308	-1.142*
	(0.893)		(0.690)	(0.681)	(0.746)	(0.704)	(0.674)
vs $L2a$	0.320	0.700		-0.314	-0.420	1.009	-0.441
	(0.907)	(0.690)		(0.699)	(0.740)	(0.758)	(0.697)
vs $L2b$	0.633	1.014	0.314		-0.107	1.322*	-0.128
	(0.926)	(0.681)	(0.699)		(0.758)	(0.757)	(0.696)
vs $L3a$	0.740	1.121	0.420	0.107		1.429*	-0.0208
	(0.948)	(0.746)	(0.740)	(0.758)		(0.819)	(0.741)
vs $L3b$	-0.689	-0.308	-1.009	-1.322*	-1.429*		-1.450*
	(0.951)	(0.704)	(0.758)	(0.757)	(0.819)		(0.742)
vs $L4$	0.761	1.142*	0.441	0.128	0.0208	1.450*	
	(0.922)	(0.674)	(0.697)	(0.696)	(0.741)	(0.742)	

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Standard errors are reported in the parentheses. Each row shows a regression with a different type being the base outcome. Session fixed effects are included in the multilogit regressions. Unidentifiable subjects are excluded.

Table 16: Multilogit regression of types on Learning Effects (2)